International Journal of Advance Research and Innovation
Vol. 2(1), Jan-Mar 2014, pp. 36-41
Doi: 10.51976/ijari.211405
www.gla.ac.in/journals/ijari

Article Info

Received:02 Feb 2014 | Revised Submission: 10 Jan 2014 | Accepted: 20 Jan 2014 | Available Online: 15 Mar 2014

Numerical Analysis of Cost Function of Electric Supply Reliability of a Standby System with Critical Human Error and Environmental Failure

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ABSTRACT

This paper presents an electric supply system consisting of two subsystem, A & B in series (1-out-of-2: F) viz. Generation system and supply respectively. The electric system fails, if any of the two subsystems A&B fails. The generation system A consists of three identical units. Initially only two units work and third unit is in standby. The standby unit is operated when any of the two units fails, through the automatic changeover, which is assumed to be perfect. The electric system goes to degraded state when only one unit of A is operable. The electric system may also fail due to critical human error and environmental failure. The failure time for the system follows exponential distribution whereas repair time assumes arbitrary distribution. Using Laplace transform various state probabilities and evaluated along with cost function. Numerical examples have been added to highlight the important results.

Keywords: *Standby Redundancy; Environmental Failure; Human Error.*

1.0 Introduction

Pandey and Jacob (1993) considered a two unit three state standby system with cold standby under the assumption that the system works in degraded state on failure of one operable unit. The standby unit becomes operable only when both the unit fails and system remains in degraded state. This assumption does not seem realistic in the present era because no one would like to compromise with the service quality. Mokadis et. al. (1997) have studied on Analysis of a two unit warm standby subject to degradation. Mokadis et. al. (1997) have discussed on Cost analysis of a two dissimilar unit cold standby redundant system subject to inspection and two types of repair. Parthasarthy (1979) has studied on Cost analysis of two unit system. Sharma and Agarwal (1996) have worked on Some reliavilty measures of a system of components sharing a common environment. Anette (2008) has studied on Cost-oriented failure mode and effects analysis. Sharma et. al. (2009) have discussed on Reliability and cost analysis of Utility company website using middleware solution by mathematical modeling. Garg and Goel (1985) have worked on Cost analysis of a system with common cause failure and two types of repair facilities. Gupta et. al. (2006) have studied on Cost benefit analysis of a three unit complex system with correlated failures and repairs. Further a perusal of reveals the fact that a lot of work has been done in this direction, by no attention has been given to the effect of environmental failure and human error on system reliability.

Keeping all these facts in view, the authors have considered a power system consisting of two subsystems A & B in series (1-out-of-2:F) viz: Generation system and supply system respectively. The power system fails if any of the two subsystems fails. The Generation subsystem A consists of three identical units. Initially only two units work and third unit is in standby. The standby unit is operated when any of the two units fails, through the automatic changeover, which is assumed to be perfect. The power system goes to degraded state when only one unit of A is operable. The electric system may also fail due to critical human error and environmental failure. The failure time for the system follows exponential distribution whereas repair time assumes arbitrary distribution. Using Laplace transform various state probabilities are evaluated along with

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cost function. Numerical examples have been added to highlight the important results.

2.0 Assumptions

1. Initially the system is good.
2. Subsystem *A* consists of three units, initially two units operate and third is in standby.
3. Subsystem *B* is a single unit.
4. System goes to degraded state when only one unit of subsystem *A* works.
5. The system fails only when any one subsystem either *A* or *B* fails.
6. The system may fail completely due to critical human error and environment failure.
7. The switching over device is assumed to be perfect.
8. The repaired units/subsystem work as new. Repair does not damage anything.

3.0 Notations

λ_A, λ_B : Failure rate of each unit of subsystem *A* and subsystem *B* respectively.

λ_H, λ_E : Failure rate system for critical human error and environmental failure respectively.

$\phi_F(x), \phi_H(y)$: General repair rates.

η : Constant repair rate of environmental failure.

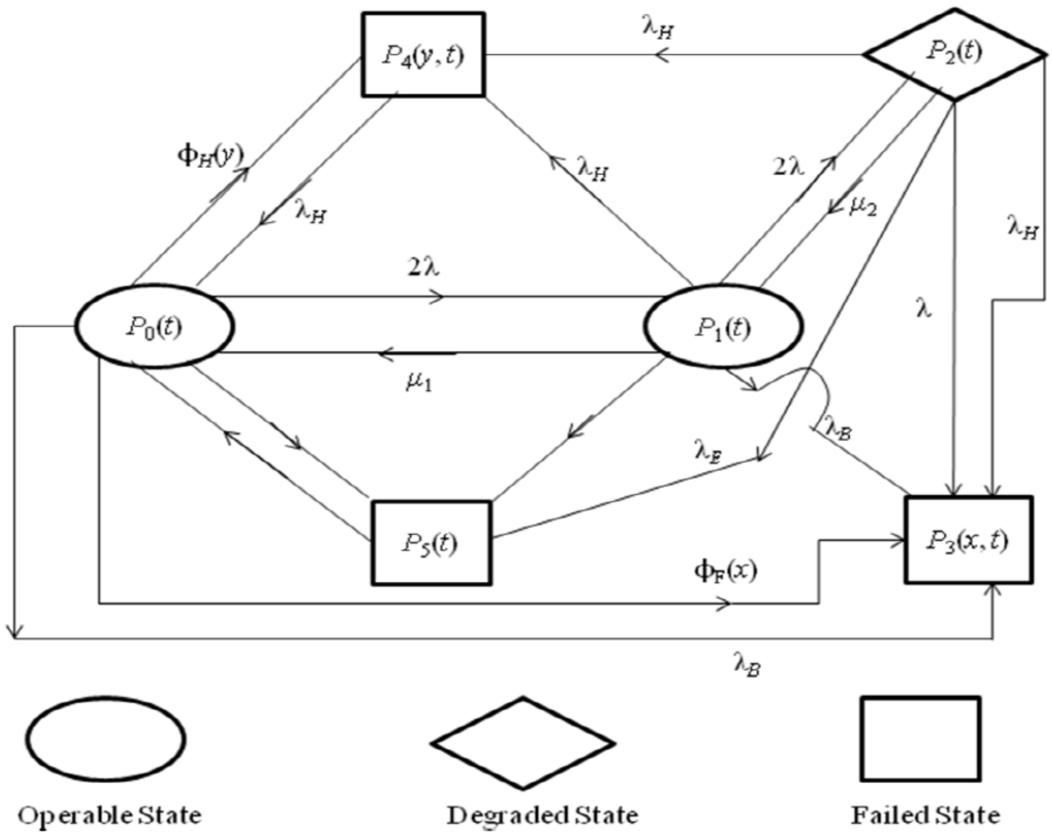
$P_i(t)$: Probability that the system is in operable state at time *t* for *i* =0, 1, 2.

μ_1 / μ_2 : Constant repair rates from state 1/2.

$P_3(x,t) / P_4(y,t)$: Probability that at time t system is in failed state and elapsed repair time lies in interval $(x, x + \Delta)/(y, y + \Delta)$.

$P_5(t)$: Probability of system being in failed state due to environmental failure.

4.0 State Transition Diagram



5.0 Formulation of Mathematical Model

By elementary probability and continuity arguments, the difference-differential equations for the stochastic process, which is continuous in time and discrete in space, are:

Taking Lapalace Transforms of equations (1) and (8) and then solving with the help of initial conditions, one may obtain.

$$\left[\frac{\partial}{\partial t}+2\lambda+\lambda_H+\lambda_B+\lambda_E\right]P_0(t)$$

$$=\mu_1 P_1(t)+\int_0^\infty P_3(x,t)\phi_F(x)dx+\int_0^\infty P_4(y,t)\phi_H(y)dy+\eta P_5(t) \quad \dots(5.1)$$

$$\left[\frac{\partial}{\partial t}+\lambda+\lambda_H+\lambda_B+\lambda_E+\mu_1\right]P_1(t)=2\lambda P_0(t)+\mu_2 P_2(t) \quad \dots(5.2)$$

$$\left[\frac{\partial}{\partial t}+\lambda+\lambda_H+\lambda_B+\lambda_E+\mu_2\right]P_2(t)=\lambda P_1(t) \quad \dots(5.3)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\phi_F(x)\right]P_3(x,t)=0 \quad \dots(5.4)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\phi_H(y)\right]P_4(y,t)=0 \quad \dots(5.5)$$

$$\left[\frac{\partial}{\partial t}+\eta\right]P_5(t)=\lambda_E\{P_0(t)+P_1(t)+P_2(t)\} \quad \dots(5.6)$$

Boundary Condition:

$$P_3(0,t)=\lambda_B\left[P_0(t)+P_1(t)+P_2(t)\right]+\lambda P_2(t) \quad \dots(5.7)$$

$$P_4(0,t)=\lambda_H\left[P_0(t)+P_1(t)+P_2(t)\right] \quad \dots(5.8)$$

Initial Conditions:

$$P_i(0)=\begin{cases}1, & if \quad i=0\\ 0, & \text{Otherwise}\end{cases} \quad \dots(5.9)$$

Taking Lapalace Transforms of equations (1) and (8) and then solving with the help of initial conditions, one may obtain.

$$P^*{}_0(s)=\frac{1}{D(s)} \quad \dots(5.10)$$

$$P_1^*(s)=\frac{2\lambda E(s)}{F(s)}.\frac{1}{D(s)} \quad \dots(5.11)$$

$$P_2^*(s)=\frac{2\lambda^2}{F(s)}.\frac{1}{D(s)} \quad \dots(5.12)$$

$$P_3^*(s)=\left[\lambda_B G(s)+\frac{2\lambda^2}{F(s)}\right]r_F(s).\frac{1}{D(s)} \quad \dots(5.13)$$

$$P_4^*(s)=\left[\lambda_H G(s)\right]r_H(s).\frac{1}{D(s)} \quad \dots(5.14)$$

$$P_5^*(s)=\left[\frac{\lambda_E G(s)}{(s+\eta)}\right].\frac{1}{D(s)} \quad \dots(5.15)$$

$$A(s)=s+2\lambda+\lambda_H+\lambda_B+\lambda_E-\lambda_H G(s)S_H(s)$$

$$B(s)=-2\mu_1\lambda\frac{E(s)}{F(s)}-\left[\lambda_B G(s)+\frac{2\lambda^3}{F(s)}\right]S_F(s)-\frac{\eta\lambda_E}{(s+\eta)}G(s)$$

$$E(s)=s+\lambda+\lambda_H+\lambda_E+\lambda_B+\mu_2$$

$$F(s)=\left[s+\lambda+\lambda_H+\lambda_E+\lambda_B+\mu_1\right]\left(s+\lambda+\lambda_H+\lambda_E+\lambda_B+\mu_2\right)-\mu_2\lambda$$

$$G(s)=\left[1+\frac{2\lambda E(s)}{F(s)}+\frac{2\lambda^2}{F(s)}\right]$$

$$r_i(s)=\frac{1-S_i(s)}{s} \qquad S_i(s)=\int_0^\infty \phi_i(r)e^{-sr-\int_0^r \phi_i(r)dr}dr$$

$$i=F, \quad r=x$$
$$i=H, \quad r=y$$
$$D(s)=A(s)+B(s)$$

6.0 Evaluation of up and Down Probabilities

Laplace Transforms of the probabilities that the system is in up (i.e., good or degraded) or down (i.e. failed states at time *t*) are as follows:

$$P_{up}^*(s)=P_0^*(s)+P_1^*(s)+P_2^*(s)$$

$$P_{up}^*(s)=\frac{G(s)}{D(s)} \quad \dots(6.1)$$

$$P_{down}^*(s)=P_3^*(s)+P_4^*(s)+P_5^*(s)$$

$$=\frac{1}{D(s)}\left[\left(\lambda_B G(s)+\frac{2\lambda_E(s)}{F(s)}+\frac{2\lambda^2}{F(s)}\right)r_F(s)+\lambda_H G(s)r_H(s)+\frac{\lambda_E}{(s+\eta)}G(s)\right] \quad \dots(6.2)$$

It will be interesting to note that

$$P_{up}^*(s)+P_{down}^*(s)=\frac{1}{s} \quad \dots(6.3)$$

7.0 Steady State Behavior

Using Able's Lemma

$$\lim_{s\to 0} sH(s)=\lim_{t\to\infty} H(t)=H \qquad (say)$$

$$P_{up} = \frac{G(0)}{D'(0)} \quad \ldots (7.1)$$

$$P_{down}(0) = \frac{1}{D'(0)}\left[\left(\lambda_B G(0) + \frac{2\lambda^3}{F(0)}\right) r_F(0) + \lambda_B G(0) r_H(0) + \frac{\lambda_E}{s+\eta} G(0)\right] \ldots (7.2)$$

Where $D'(0) = \left(\frac{d}{ds}D(s)\right)_{s=0}$

$G(0) = \left[G(s)\right]_{s=0}$

$E(0) = \left[E(s)\right]_{s=0}$

And $F(0) = \left[F(s)\right]_{s=0}$

8.0 When Repair Follows Exponential, Distribution

Setting $S_F(s) = \frac{\phi_F}{s+\phi_F}$, $S_H(s) = \frac{\phi_H}{s+\phi_H}$ one may obtain

$$P_0^*(s) = \frac{1}{D_1(s)} \quad \ldots (8.1)$$

$$P_1^*(s) = \frac{2\lambda E(s)}{F(s)} \cdot \frac{1}{D_1(s)} \quad \ldots (8.2)$$

$$P_2^*(s) = \frac{2\lambda^2}{F(s)} \cdot \frac{1}{D_1(s)} \quad \ldots (8.3)$$

$$P_3^*(s) = \left[\frac{2\lambda^3}{F(s)}\right] \cdot \frac{1}{(s+\phi_F)D_1(s)} \quad \ldots (8.4)$$

$$P_4^*(s) = \lambda_H G(s) \cdot \frac{1}{(s+\phi_H)D_1(s)} \quad \ldots (8.5)$$

$$P_5^*(s) = \frac{\lambda_E G(s)}{(s+\eta)} \cdot \frac{1}{D_1(s)} \quad \ldots (8.6)$$

Making use of equations (8.1) to (8.6), one may obtain.

$$P_{up}(s) = \frac{G(s)}{D_1(s)} \quad \ldots (8.7)$$

9.0 Reliability

Taking all repair rates equal to zero, one may obtain.

$$R(s) = \sum_{i=0}^{2} P_1(s)$$

$$R(s) = \frac{1}{(s+a)} + \frac{2\lambda}{(s+b)^2} \quad \ldots (9.1)$$

Where $D_1(s) = A_1(s) + B_1(s)$

$$A_1(s) = s + 2\lambda + \lambda_H + \lambda_B + \lambda_E - \lambda_H G(s)\frac{\phi_H}{(s+\phi_H)}$$

$$B_1(s) = -2\mu_1\lambda\frac{E(s)}{F(s)} - \left[\lambda_B G(s) + \frac{2\lambda^3}{F(s)}\right]\frac{\phi_F}{(s+\phi_F)} - \frac{\eta\lambda_E}{(s+\eta)}G(s)$$

By inversion process one may obtain reliability as

$$R(t) = e^{-at} + 2\lambda t e^{-bt} \quad \ldots (9.2)$$

10.0 Expected Cost

If α_1 & α_2 be the revenue cost and service cost per unit time then expected cost $H(t)$ will be obtained by

$$H(t) = \alpha_1 \int_0^t R(t)dt - \alpha_2 t$$

$$H(t) = \alpha_1 \left\{\frac{1}{a}(1-e^{-at}) + \frac{2f}{b^2}(1-e^{-bt} - bte^{-bt})\right\} - \alpha_1 t \quad \ldots (10.1)$$

Where $a = 2\lambda + \lambda_H + \lambda_B + \lambda_E$

$b = \lambda + \lambda_H + \lambda_B + \lambda_E$

11.0 Variance of Time to Failure:

Variance of time to failure is obtained by

$$\sigma^2 = -2\lim_{s\to 0} dR(s) - (MTTF)^2 \quad \ldots (11.1)$$

Where $MTTF = \lim_{x\to 0} R(s)$

Using equation (9.2) in equation (11.1), one may obtain.

$$\sigma^2 = 2\left(\frac{1}{a^2} + \frac{4\lambda}{b^3}\right) - \left(\frac{1}{a} + \frac{2\lambda}{b^2}\right)^2 \quad \ldots (11.2)$$

12.0 Results and Discussion:

Numerical calculations have been carried out for the effect of constant environmental failure on system for different values of parameters and are displayed in Figures-(1) to (3).

12.1 Reliability analysis

The Reliability for system is plotted in figure – (1) at $\lambda = 0.01$, $\lambda B = 0.02$ and $\lambda B = 0.02$ and different value of failure environment λE, it is noticed that the reliability of the system decreases with increase of time for various values of environmental failure.

12.2 Cost analysis

The Expected Cost per unit time for system is plotted in figure – (2) at $\alpha 1 = 1$, λ = 0.01, λH = 0.05 and λE = 0.03 and different value of service cost α2, The Graph, "Expected Cost vs Time" indicates that the cost function increases initially and at last becomes steady.

12.3 Variance of time to failure:

The Variance of time to failure for system is plotted in figure – (3) at λ = 0.01, λH = 0.01 and λB = 0.02. The Graph "Variance of time to failure vs Environmental failure" discloses the fact that variance of time to failure decreases as value of environmental failure increases.

Fig. - 1 : Reliability of a system for different value of λ_B

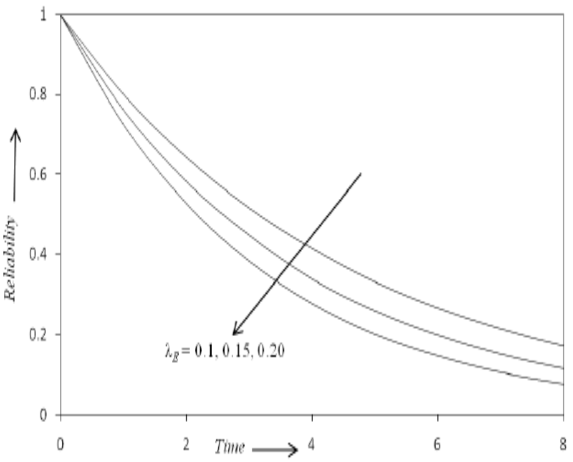


Fig. - 2: Expected Cost for different value of α_2

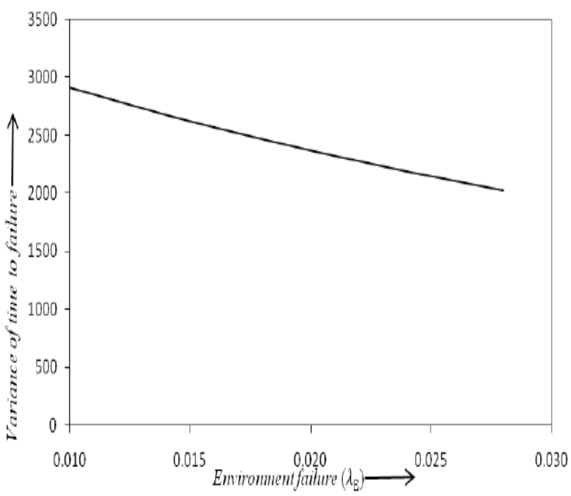
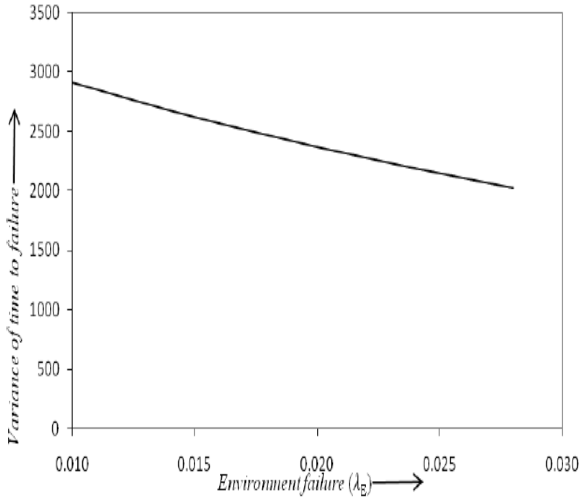


Fig.- 3: The Variance of time to failure vs Environment failure λ_E.



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