Delhi Business Review Vol. 22, No. 2 (July - December 2021)

DELHI BUSINESS REVIEW

An International Journal of SHTR

Journal Homepage: https://www.delhibusinessreview.org/Index.htm
https://www.journalpressindia.com/delhi-business-review

Geometric Brownian Motion & Nifty 50 Index: A Confirmation from National Stock Exchange

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ARTICLE INFO

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Article history:
Received 02 September 2021
Revised 11 September 2021
24 September 2021
Accepted 10 October 2021

Keywords:
Geometric Brownian motion,
K-S test,
Q-Q plot
MAPE

ABSTRACT

Purpose: *The study's major goal is to create a model for projecting the Nifty 50 closed stock price's short-term return distribution.*
Design/Methodology/Approach: *A stochastic process, has been followed in the current study. Geometric Brownian Motion has been used for managing investment risk. The article has used Geometric Brownian Motion for managing investment risk. Nifty 50 closed prices have been used for this study.*
Findings: *The Kolmogorov-Smirnov (K-S) statistic of Nifty 50 closed stock price for 226 days from the 13th July, 2020 to the 9th June, 2021 is 0.253113. As calculated K-S stat 0.253113 is less than table value 1.35810, the series is normally distributed. The Q-Q plot shows that with the dots forming a fairly straight line and the data set is normal distribution. The goal of the selection procedure between all simulations is to guarantee that the forecast data in the return distribution have the same structure and trend as the Nifty 50 closing stock price.*
Research Limitations/Implications: *It considers the data from the 13th July, 2020 to the 9th June, 2021 is used in this article to determine whether the data are normally distributed and viable to estimate for future stock prices.*
Practical Implications: *The technician's major purpose is to make profit at the expense of other investors. He should find high volatile stocks in any holding period for effective forecast. Regulatory bodies in the market should focus on long-term stability by accumulating reserves that can cushion off the consequences of illiquidity, especially in the period of significant volatility.*
Originality/Value:
Geometric Brownian Motion model is used to forecast. It is more accurate model in compare to other models. Investors can take shelter of this model for their investment's decision. K-S test & Q-Q plot method are used for the normality test. Simulation is used by using GBM equation. MAPE is designed to decide the forecast accuracy.

DOI: 10.51768/dbr.v22i2.222202110

Introduction

Financial markets are very important market. It is place where we can play with investment. Companies can have their required fund in this market and on the other hand people of society can find this market for their investment. In one sense you can imagine financial market as a stock market as because it is an important part of financial market.

Investors can get high return from their investment in stock market. For capital appreciation it is a good place to take funds from this market. Stock market is a place where people around the world could swim according to their goal. In stock market there is high risk associated with return. So, investors should have proper information to minimize risk. Stock price not only shows all information regarding stocks but also reflects potential performance of companies associated with the stocks. As demand is indirectly related with the growth of an economy stock market plays a vital function for the economic growth of a country. Each and every impulse plays an important role in the stock market. COVID-19 as a negative impulse could affect the Indian stock market. Performance of the stock market could affect the performance of the economy.

If the stock market is more volatile due to the changes in prices of securities, then it is very difficult to forecast the stock price and investors may face loss from their investment. To get better understanding about the stock price in the short period Geometric Brownian Motion could play an important role in this respect. In this article we have tried to catch the volatility in Nifty 50 stock price by using the Geometric Brownian Motion model.

Review of Literature

most effective. Simulating the values of stocks, derivatives, and options is something that financial advisors, academics, investors, financial managers business analysts, and are interested in doing in order to make significant investment and financial plans.

Agbam & Isukul (2020) in their study, a geometric Brownian motion is used to describe stochastic stock price changes. The GBM with drift assumptions (continuity, normalcy, and Markov tendency) were explored using historical closing prices of 10 listed companies in the 8 sectors of the Nigeria Stock Exchange for four years (2015–2018). The most actively traded equities are used to make the stock pick. To test the model's validity, the anticipated prices were compared to the actual prices. The results demonstrate that there are some actual stock prices outside of the trajectory realization in the simulation, which could be due to Geometric Brownian motion. The model's predictive power decreases as the assessed time frame lengthens, as evidenced by the larger mean absolute percentage error value.

AEFF (1995), in his article tried to focus on the technical analysis and fundamental analysis for forecasting stock prices. Technical analysis basing on historical information tries to forecast the stock price. On the other hand, fundamental analysis uses the intrinsic value depending on the earning potential of stocks at any point in time to have an idea if the stock is overvalued or undervalued. Therefore, from the Fama's article it is found that forecasting is important for predicting upcoming security price.

Farida Agustini et al. (2018), in their article the mathematical understanding for stock price prediction using the geometric Brownian Motion model started by computing the rate of return, then estimates the value of volatility and drift, obtains the stock price forecast, calculated the forecast MAPE, calculated the stock expected price, and calculated the 95 percent confidence level. The outcome research demonstrated that the geometric Brownian motion model was the most accurate prediction technique. It had been proven with an anticipated MAPE value of less than 20%

Reddy & Clinton (2016), in their article to stimulate the path of stock price used the geometric Brownian motion (GBM) technique, and it was also tested if stock price (simulated) lined up with returns that is actual. Australian companies registered on the S&P/ASX 50 Index had been used for the study. Daily basis stock prices over the period 01.01.2013 – 31.12.2014 was collected from Thomson One data base. The findings showed that over the study period simulated stock price using GM method traveled in a motion in the same as actual security prices were a tiny larger than 50%.

Ladde & Wu (2009) used classical model building techniques to create modified linear GBM models. They examined the accuracy of the existing GBM model using stock price data and basic statistical techniques. They next showed how the modified GBM model developed under various data splitting and without and with skips. They used a Monte Carlo approach to compare the built and GBM models. The findings imply that data splitting improves performance, with models with jumps doing significantly better than those without.

Marathe & Ryan (2005) provide how to determine if a given time series follows the GBM method. They also look at approaches for removing seasonal variation from a time series, which they believe is significant because the GBM process ignores cyclical and seasonal influences. The time series for utilization of established services fitted the criteria for a GBM mechanism in all four industries analyzed, while data from the expansion of emergent services did not.

Alhagyan et al. (2017) in their article looked at approaches for forecasting future adjusted Stamp; P 500 prices. For improved investment decisions, use geometric Brownian motion (GBM) and geometric fractional Brownian motion (GFBM). To discover the proper volatility measurement that could produce forecast value that nearly resembled real stock price movement, four types of algorithms were used. The mean absolute percentage error was used to evaluate forecasting algorithms (MAPE). The results revealed that all forecasting methods had good accuracy, with all MAPE being less than 10%. The best forecasting approach was GFBM with stochastic volatility, which followed the fractional Ornstein – Uhlenbeck (FOU) process.

Sengupta (2004), in his book "Financial Modeling using Excel and VBA" mentioned that by simulating the security price it is possible to generate a path that is followed by a security on the basis of the historical information.

This paper's contribution despite a wealth of literature on the application and variations of the GBM model, there appears to be a dearth of study on the model's forecasting accuracy, and hence the validity of the GBM assumption. The sample in this work was chosen to represent a market with little GBM research. The research examines the GBM assumption's validity throughout a variety of holding periods. This is a short-term test of the GBM model's validity.

Objectives of the Study

The study's major goal is to create a model for projecting the Nifty 50 closed stock price's short-term return distribution. The following goals are the focus of this research.

- Using the Kolmogorov-Smirnov normality test and the Q-Q plot technique, determine if the observed Nifty 50 closing stock price is normal or not.
- Create a Geometric Brownian Motion model to predict the Nifty 50 closed stock price or distribution of return distribution in the future.
- Simulated the predicted data to find the best simulation for replicating the reported data's movement. Computes the Mean Absolute Percentage Error (MAPE) to assess the model's forecast accuracy.

Research Methodology

Brownian Motion:

A stochastic process, $\{W_t: 0 \leq t \leq \infty\}$, is a standard Brownian motion if

1. $W_0 = 0$
2. It has continuous sample path
3. It has independent, normally-distributed increments.

$$W_t - W_s \sim N(0, t-s)$$

Here mean is zero and variance is t-s.

The condition of independent increments means that if $0 \leq S_1 \leq t_1 \leq S_2 \leq t_2$ then $(W_{t1} - W_{s1})$ and $(W_{t2} - W_{s2})$ are independent random variables.

N- Dimensional Brownian Motion:

An n-dimensional process $W_t\{W_t^1, \ldots, W_t^n\}$ is a standard n-dimensional if each W_t^i is a standard Brownian motion and the W_t^is are all independent of each other.

Conversion of Random Walk into Brownian Motion:

Divide the interval t into n parts each of size t/n.

Each increment would be $R_i = \sqrt{\frac{t}{n}}$

The total increment over $S_i = \sum_{i=n}^{n} R_i$

As Martingale process is followed we have

$E[S_i] = 0 \quad \& \, E[R_i] = 0$

$E[R_i^2] = t/n$

$E[S_i^2] = E[(R_1 + \ldots + R_i)(R_1 + \ldots + R_i)]$

When $i \neq j \quad E[R_i R_j] = 0$ *because they are uncorrelated.*

Now,

$E[S_i^2] = E[(R_1^2 + \ldots + R_i^2) = i\left(\frac{t}{n}\right)$

$E[S_n^2] = E[(R_1^2 + \ldots + R_i^2) = t$

Let $n \to \infty$ *on a random walk to get Brownian motion*

Limit as $n \to \infty \Rightarrow X(t)$ a Brownian motion

$E[X(t)] = 0 \quad \& \, E[R_i] = 0$

$E[(X(t))^2] = t$

This is Markovian, finite, continuous, a Martingale, normal (0,t)

Now Wiener or Brownian Process with Drift:

$$dx = adt + bdW(t)$$

In the above equation total change in x has two parts. One is something that grows with time and another is variable part or noise part that could go up or could go down.

$dx = adt$ can be integrated to

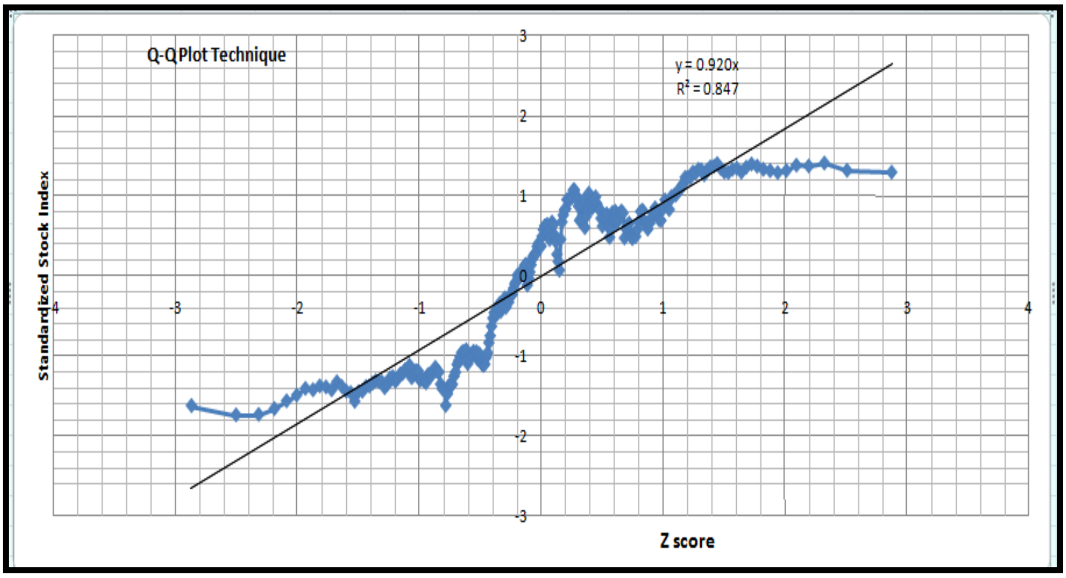

Fig 1: Q-Q Plot Method
Source: Authors Calculations

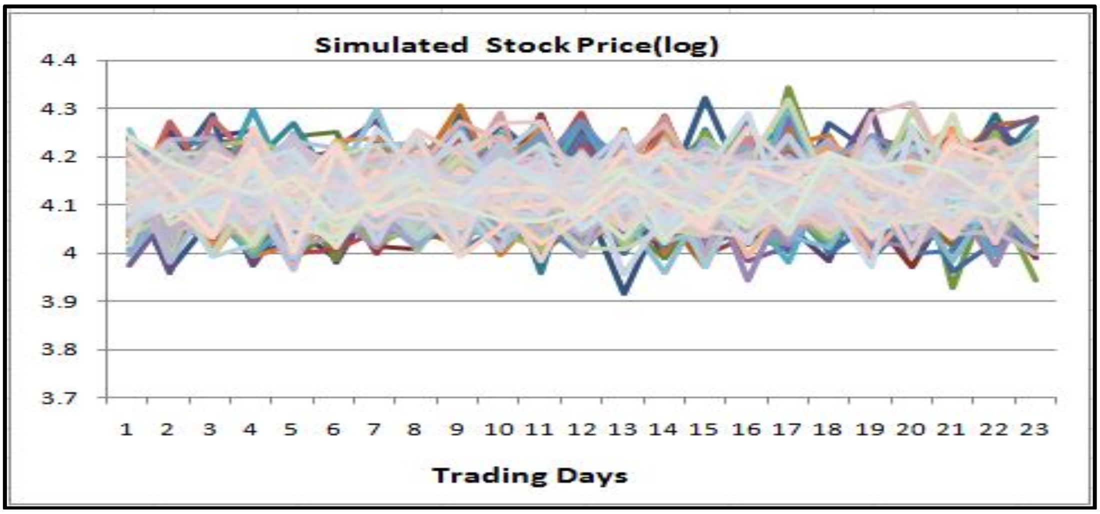

Figure 2: Simulated Stock Price
Source: Authors Calculations

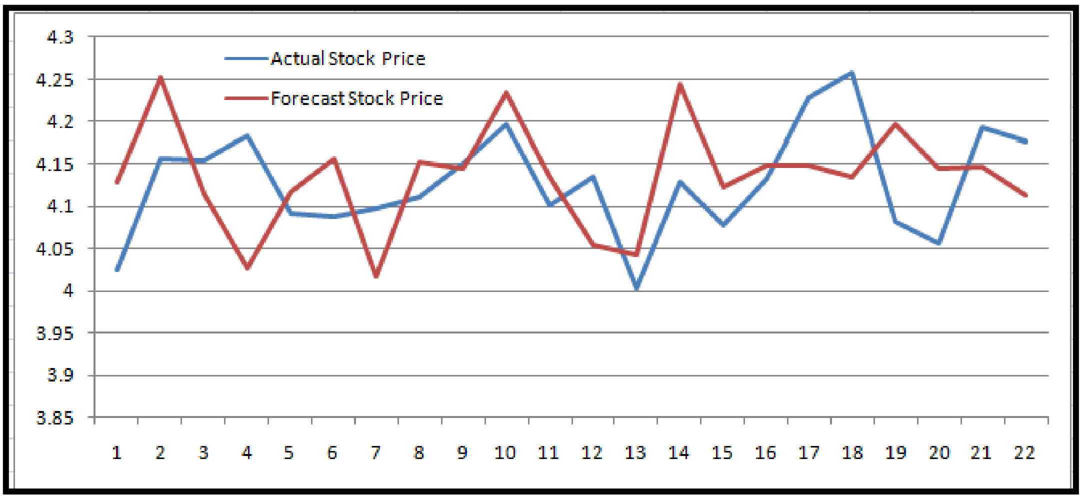

Figure 3: Assessment between Actual and Forecast Stock Price
Source: Authors Calculations

$x = x_0 + at$
where x_0 is the initial value and if the time period is t, the variable increases by at. bdz accounts for the noise or variability to the path followed by x. The amount of this noise or variability is b times

Empirical Findings and Analysis:
The data from the 13th July, 2020 to the 9th June, 2021 is used in this article to determine whether the data are normally distributed and viable to estimate for future stock prices, and the data from the 10th July, 2021 to the 9th July, 2021 will be utilized as validation to equate with the estimated data.

Kolmogorov-Smirnov Test and Q-Q Plot Technique
"The Kolmogorov-Smirnov (K-S) statistic is used to measure how well the distribution of a random sample $(X_1, X_2, \ldots\ldots X_n)$ agrees with a theoretical distribution
It is defined as

$$D_n = {}^{sup}_{x}|F_n(x) - F(x)|$$

Where n is the sample size, $F_n(x)$ be the empirical distribution function of $(X_1, X_2, \ldots\ldots X_n)$ and F(x) be the cumulative distribution function of a pre-specified theoretical distribution, under the null hypothesis that the sample $(X_1, X_2, \ldots\ldots X_n)$ comes from F(x) or the data are normally distributed.

Rate	Drift (µ)	Volatility (σ)
Actual	4.19	0.0017
Forecast	4.13	0.0411

Table 1: Comparison between actual and forecast drift and volatility
Source: Authors Calculations

The Kolmogorov-Smirnov (K-S) statistic of Nifty 50 closed stock price for 226 days from the 13th July, 2020 to the 9th June, 2021 is 0.253113. Form the table it is found that K-S statistic K – S statistic ≥ 0.05. As calculated K-S stat 0.253113 is less than table value 1.35810, the series is normally distributed. We cannot reject null hypothesis i.e., Null Hypothesis: Normally distributed.

Figure 1 shows a visual representation of the Q-Q plot method, with the dots forming a fairly straight line and the data set approaching normal distribution.

Forecasting with Brownian motion:
Figure 2 shows a simulation of the Nifty 50 closing stock price utilizing Geometric Brownian Motion for a period of 23 days, from July 10th to July 9th, 2021, using the described methods.

For stock price forecasting, a simulation is run using 226 realizations of a path that might be derived from Geometric Brownian motion with twenty-three repetitions. Table 1 shows the actual and anticipated drift and volatility based on actual and forecast values.

The closest forecast simulation with actual value of Nifty 50 closed stock price is selected with more precise value of drift and volatility and presented in Figure – 3. The goal of the selection procedure between all simulations is to guarantee that the forecast data in the return distribution have the same structure and trend as the Nifty 50 closing stock price. This type of simulated return distribution can be utilized to manage investment risk and enable investors to make more informed incantations.

The correlation coefficient between the simulated stock price of the return distribution and the real stock price is positive (=0.34), indicating that there is a connectivity between the actual stock price and the anticipated stock price in the long term.

Accuracy of Forecasting:
Due to its advantages of scale independence and interpretability the Mean Absolute Percentage Error (MAPE), also known as Mean Absolute Percentage Deviation (MAPD), is one of the most prominent measures of forecast accuracy. When the actual values are zero or close to zero, it gives infinite or undefined values. The MAPE produces exceptionally huge percentage errors or outliers if the real value is very small (less than one). It proposes a solution to this problem by removing outliers with real values less than one or Absolute Percentage Error (APE) values more than the MAPE plus three standard deviations.

The Geometric Brownian Motion model is highly accurate for forecasting the Nifty stock price since the MAPE is 0.018, i.e., the MAPE is less than 2%. The Geometric Brownian Motion model is ideal for short-term forecasting due to the random behaviour of stock prices.

Conclusion and Recommendations

Conclusion
In this research article Geometric Brownian Motion has been used for managing investment risk. Nifty 50 closed prices have been used for this study. The Kolmogorov-Smirnov (K-S) statistic of Nifty 50 closed stock price for 226

days from the 13th July, 2020 to the 9th June, 2021 is 0.253113. As calculated K-S stat 0.253113 is less than table value 1.35810, the series is normally distributed. The Q-Q plot shows that with the dots forming a fairly straight line and the data set is normal distribution. The goal of the selection procedure between all simulations is to guarantee that the forecast data in the return distribution have the same structure and trend as the Nifty 50 closing stock price. This type of simulated return distribution can be utilized to manage investment risk and enable investors to make more informed incantations. The Geometric Brownian Motion model is highly accurate for forecasting the Nifty stock price since the MAPE is 0.018. Therefore, The Geometric Brownian Motion model is ideal for short-term forecasting due to the random behaviour of stock prices.

Recommendations

The technician's major purpose is to make profit at the expense of other investors. He should find high volatile stocks in any holding period for effective forecast. Regulatory bodies in the market should focus on long-term stability by accumulating reserves that can cushion off the consequences of illiquidity, especially in the period of significant volatility.

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