

Article Info

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Impact of Phasor Measurement Unit on The State Estimation of Large Power System

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ABSTRACT

The conventional iteration processes of power flow measurement of a system are bulky in nature. The new technique for measurement of bus voltage and power flow is Phasor Measurement Unit. Usually, the concept of full weighted least square state estimator is a nonlinear technique, but with conventional process and PMUs it may improve the accuracy of the measurement without doing a bulky iteration procedure. In this paper the way of formation of measurement by using Full weighted least square state estimation and PMU device with conventional method will be investigated. A number of cases are tested by use of PMUs and their effect on variables accuracy on system bus current and power flows are demonstrated. The assessment of parameter obtained on IEEE 6 bus and IEEE 9 bus system will be discussed.

Keywords: Conventional Iteration; Full Weighted Least Square (WLS) state technique; State Estimation; Phasor Measurement Units.

1.0 Introduction

Phasor Measurement Unit deals with the electrical waves on an electric grid to measure the health of the power system. A phasor is a multifaceted quantity that represents both the magnitude and phase angle of the sine waves found in electric signals. The phasor measurements that occur at the same instance are called "Synchrophasor", as the PMU devices that consent to their measurement. In power system engineering, traditionally we use number of devices such as Synchro-scope, Isolator, Ammeter, Circuit Breaker, Relay, CT & PT, Voltmeter, Wattmeter, Compensator devices, etc. for measurement purposes on every station. The recently developed PMU [3][6][11][12][13][14] will help in deciding to stall such devices at proper location for:

- Accurate and comprehensive planning.
- Better congestion tracking,
- Visualization and advanced warning systems,

The conventional processes of measurement are too iterative and bulky in nature for measurement of power flow and voltage on a bus of system. The full weighted least square state technique [1][4][12] [13] is a nonlinear equation

but with first order Taylor series become a linear equation. Some research work is already conducted in formulation of a relation between full weighted least square state and PMUs. The natural approach for parameter measurement will treat PMU as additional computational burden on measurement and calculation. The problem of finding optimal placement of PMU devices for state estimation of power system is investigated. This paper reflects the accuracy of measurement with or without using PMU on state estimation parameters. In case 1, the state estimation of system by conventional process without using any PMU [5] device, but in case P, measurement with the use of PMU is discussed.

2.0 FULL WEIGHTED LEAST SQUARE STATE ESTIMATION METHOD

Let us consider the set of measurements given by the vector z as:

$$z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ \vdots \\ z_m \end{bmatrix} = \begin{bmatrix} h_1(x_1, x_2, x_3, \dots, x_n) \\ h_2(x_1, x_2, x_3, \dots, x_n) \\ h_3(x_1, x_2, x_3, \dots, x_n) \\ \vdots \\ h_m(x_1, x_2, x_3, \dots, x_n) \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ \vdots \\ e_m \end{bmatrix} = h(x) + e \quad (1)$$

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Where:

$$h^T = [h_1(x), h_2(x), h_3(x), \dots, h_m(x)] \quad (2)$$

$h_i(x)$ is the nonlinear function relating measurement i to the state vector x

$x^T = [x_1, x_2, x_3, \dots, x_n]$ is the system state vector

$e^T = [e_1, e_2, e_3, \dots, e_m]$ is the vector of measurement errors.

The WLS estimator [1][7] will minimize the following objective function:

$$J(x) = \sum_{i=1}^m \frac{(z_i - h_i(x))^2}{R_{e_i}} = [z - h(x)]^T R^{-1} [z - h(x)] \quad (3)$$

At the minimum value of the objective function, the first-order optimality conditions have to be satisfied. These can be expressed in compressed form as follows:

$$g(x) = \frac{\partial J(x)}{\partial x} = -H^T(x) R^{-1} [z - h(x)] = 0 \quad (4)$$

The non-linear function $g(x)$ can be expanded into its Taylor series [1][4][5][7] around the state vector x^k neglecting the higher order terms.

$$g(x) = g(x^k) + G(x^k)(x - x^k) + \dots = 0 \quad (5)$$

An iterative solution scheme known as the Newton method is used to solve above equation:

$$x_{k+1} = x_k - [G(x_k)]^{-1} \cdot g(x_k) \quad (6)$$

where, k is the iteration index and x_k is the solution vector at iteration k . $G(x)$ is called the gain matrix and it expressed by:

$$G(x) = \frac{\partial g(x^k)}{\partial x} = H^T(x^k) R^{-1} H(x^k) \quad (7)$$

$$g(x^k) = -H^T(x^k) R^{-1} [z - h(x^k)] \quad (8)$$

Generally, the gain matrix is quite sparse and decomposed into its triangular factors. At each iteration k , the following sparse linear set of equations are solved using forward/backward substitutions, where $\Delta x^{k+1} = x^{k+1} - x^k$:

$$[G(x^k)] \Delta x^{k+1} = H^T(x^k) R^{-1} [z - h(x^k)] = H^T(x^k) R^{-1} \Delta e^k \quad (9)$$

These iterations are going on until the maximum variable

difference satisfies the condition, 'Max $|\Delta x^k| < \epsilon$ '.

3.0 CONVENTIONAL METHOD

The conventional method [1][4][6] of measurement is basically consider relation of power injection or power flow with respect to line current and line voltage are as

$$I_y = \sqrt{(g_y^2 + b_y^2)} (V_i^2 + V_j^2 - 2V_i V_j \cos \theta_{ij}) - \frac{\sqrt{P_y^2 + Q_y^2}}{V_i} \quad (10)$$

The Real and Reactive Power injection at bus i can be expressed as,

$$P_i = |V_i| \sum_{j=1}^N |V_j| (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) \quad (11)$$

$$Q_i = |V_i| \sum_{j=1}^N |V_j| (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) \quad (12)$$

The Real and Reactive Power Flow from bus i to bus j are as.

$$P_{ij} = |V_i|^2 (g_{ij} + g_{ji}) - |V_i| |V_j| (g_{ij} \cos \theta_{ij} + b_{ij} \sin \theta_{ij}) \quad (13)$$

$$Q_{ij} = -|V_i|^2 (b_{ij} + b_{ji}) - |V_i| |V_j| (g_{ij} \sin \theta_{ij} - b_{ij} \cos \theta_{ij}) \quad (14)$$

So the structure of the measurement of Jacobian H will be as

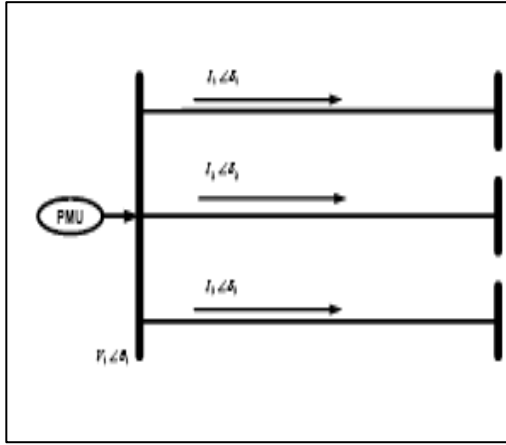
$$H = \begin{bmatrix} \frac{\partial P_{inj}}{\partial \theta} & \frac{\partial P_{inj}}{\partial V} \\ \frac{\partial P_{flow}}{\partial \theta} & \frac{\partial P_{flow}}{\partial V} \\ \frac{\partial Q_{inj}}{\partial \theta} & \frac{\partial Q_{inj}}{\partial V} \\ \frac{\partial Q_{flow}}{\partial \theta} & \frac{\partial Q_{flow}}{\partial V} \\ \frac{\partial I_{mag}}{\partial \theta} & \frac{\partial I_{mag}}{\partial V} \\ 0 & \frac{\partial V_{mag}}{\partial V} \end{bmatrix} \quad (15)$$

4.0 WLS with Conventional Method

A PMU will measure multiple current with one voltage phasors. The transmission line normally formed as pie network due to their benefit on system parameters.

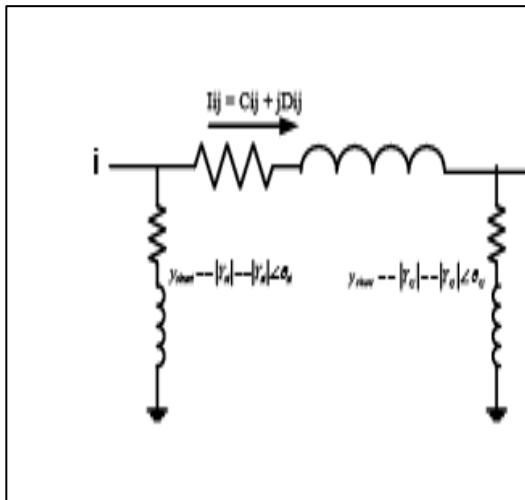
Fig. 1 shows a 4-bus system example which has single PMU at bus 1. It has one voltage phasor measurement and three current phasor measurements, namely $V_1 \angle \theta_1$, $I_1 \angle \delta_1$, $I_2 \angle \delta_2$ and $I_3 \angle \delta_3$

Figure 1: Single PMU Measurement Model



If we define y as the series admittance and y_{shunt} as the shunt admittance, current phasor measurements can be written in rectangular coordinates as shown in Fig 2.

Figure 2: Transmission Line Model



The expressions for Cij and Dij are:

$$C_{ij} = |V_i Y_{ij}| \cos(\delta_i + \theta_{ij}) + |V_j Y_{ji}| \cos(\delta_j + \theta_{ji}) - |V_i Y_{ij}| \cos(\delta_i + \theta_{ij}) \quad (16)$$

$$D_{ij} = |V_i Y_{ij}| \sin(\delta_i + \theta_{ij}) + |V_j Y_{ji}| \sin(\delta_j + \theta_{ji}) - |V_i Y_{ij}| \sin(\delta_i + \theta_{ij}) \quad (17)$$

where, the state vector is given as:

$$x = [V_1 \angle \theta_1, V_2 \angle \theta_2, V_3 \angle \theta_3, \dots, V_N \angle \theta_N]^T \quad (18)$$

The ingress of the measurement of Jacobian H corresponding to the real and reactive parts of the current phasors are as:

$$\frac{\partial C_{ij}}{\partial V_i} = |Y_{ij}| \cos(\delta_i + \theta_{ij}) - |Y_{ij}| \cos(\delta_i + \theta_{ij}) \quad (19)$$

$$\frac{\partial C_{ij}}{\partial V_j} = |Y_{ji}| \cos(\delta_j + \theta_{ji}) \quad (20)$$

$$\frac{\partial C_{ij}}{\partial \delta_i} = -|V_i Y_{ij}| \sin(\delta_i + \theta_{ij}) + |V_j Y_{ji}| \sin(\delta_j + \theta_{ji}) \quad (21)$$

$$\frac{\partial C_{ij}}{\partial \delta_j} = -|V_j Y_{ji}| \sin(\delta_j + \theta_{ji}) \quad (22)$$

$$\frac{\partial D_{ij}}{\partial V_i} = |Y_{ij}| \sin(\delta_i + \theta_{ij}) - |Y_{ij}| \sin(\delta_i + \theta_{ij}) \quad (23)$$

$$\frac{\partial D_{ij}}{\partial V_j} = |Y_{ji}| \sin(\delta_j + \theta_{ji}) \quad (24)$$

$$\frac{\partial D_{ij}}{\partial \delta_i} = |V_i Y_{ij}| \cos(\delta_i + \theta_{ij}) - |V_j Y_{ji}| \cos(\delta_j + \theta_{ji}) \quad (25)$$

$$\frac{\partial D_{ij}}{\partial \delta_j} = |V_j Y_{ji}| \cos(\delta_j + \theta_{ji}) \quad (26)$$

The measurement vector z contains δ , Cij, Dij as well as the power injections, power flows and voltage magnitude measurements.

$$z = [P_{inj}^T, Q_{inj}^T, P_{flow}^T, Q_{flow}^T, |V|^T, \delta^T, C_{ij}^T, D_{ij}^T]^T \quad (27)$$

Generally, measurements obtained from PMUs are more precise and accurate as compared to the conventional measurements. Therefore, measurements done with the help of PMUs are

expected to generate more precise and accurate result as estimated by conventional methods.

5.0 State Estimation with Pmus

The state vector and measurement data can be expressed in rectangular coordinates. The voltage measurement ($V = |V| \angle \theta$), can be expressed as ($V = E + jF$), and the current measurement can be expressed as ($I = C + jD$). Where ($g_{ij} + jb_{ij}$) is the series admittance of the line and ($g_{si} + jbsi$) is the shunt admittance of the transmission line.

Line current flow I_{ij} can be expressed as a linear function of voltages. ($V = |V| \angle \theta$).

$$I_{ij} = [(V_i - V_j) \times (g_{ij} + jb_{ij})] + [V_i \times (g_{si} + jbsi)] - V_j \times (g_{si} + jbsi) - V_j \times (g_{ij} + jb_{ij}) \quad (28)$$

The measurement vector z is expressed as $z = h(x) + e$, (where x is a state vector, $h(x)$ is a linear equations matrix, and 'e' is an error vector). In rectangular coordinates:

$$z = (Hr + jHm)(E + jF) + e \quad (29)$$

where, $H = Hr + jHm$, $x = E + jF$ and $z = A + jB$.

A and B are expressed by:

$$A = Hr \times E - Hm \times F \quad (30)$$

$$B = Hm \times E + Hr \times F \quad (31)$$

In matrix form,

$$\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} Hr & -Hm \\ -Hm & Hr \end{bmatrix} \begin{bmatrix} E \\ F \end{bmatrix} + e \quad (32)$$

Then, the estimated value $\hat{x} = \hat{E} + j\hat{F}$ can be obtained by solving the linear equation below:

$$\Delta \hat{x} = (H^T R^{-1} H)^{-1} H^T R^{-1} \Delta z = G^{-1} H^T R^{-1} \Delta z \quad (33)$$

If we define the linear matrix H_{new} as

$$H_{new} = \begin{bmatrix} Hr & -Hm \\ -Hm & Hr \end{bmatrix},$$

then the above equation can be rewritten by:

$$\hat{x} = \begin{bmatrix} \hat{E} \\ \hat{F} \end{bmatrix} = (H_{new}^T R^{-1} H_{new})^{-1} H_{new}^T R^{-1} \begin{bmatrix} A \\ B \end{bmatrix} \quad (34)$$

Therefore, the equation for rectangular formed variable $\hat{x}$ can be given by the rectangular forms of H matrix and z vector.

In respect of the system accuracy and reliability, PMU can deliver more precise measurement data. Several cases to be tested with PMUs added to the conventional measurement set. The simulations and analysis of different cases are as shown in Table 1 are done with several IEEE bus systems in the next section.

Table 1: Different cases of PMU addition in IEEE System

Cases	Measurements
1	Conventional with No PMUs
P	Only PMUs

6.0 Simulation Results

For investigate the system accuracy with or without PMU on system variables, some cases are tested with the help of MATLAB software. The testing parameters are available on conventional process with or without PMU.

Table 2: PMU Locations for Each IEEE System

Type of System	PMU locations at Bus
IEEE 6 System	Bus 2
IEEE 9 System	Bus 2

The circuit diagram will be shown as in Figure 3 and Figure 4 for IEEE 6 bus and IEEE 9 bus system respectively.

Figure 3: IEEE 6 Bus System

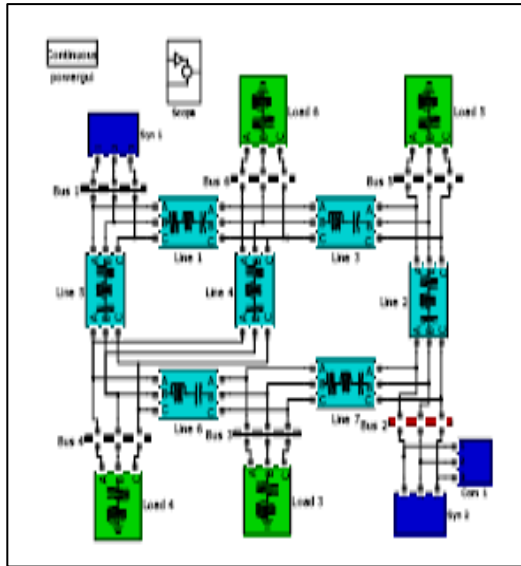
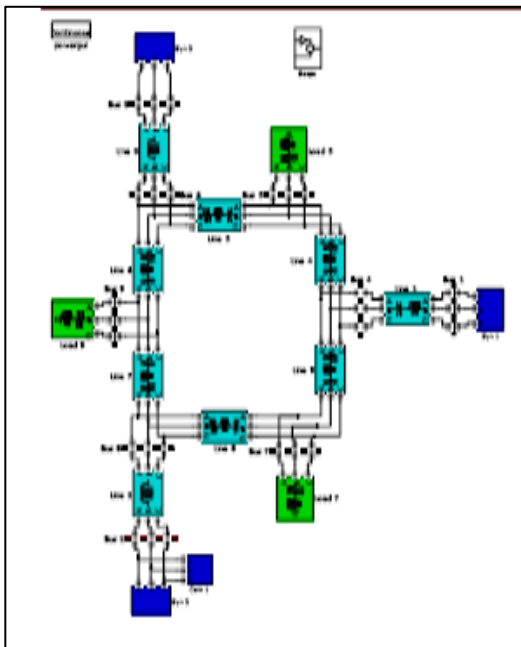
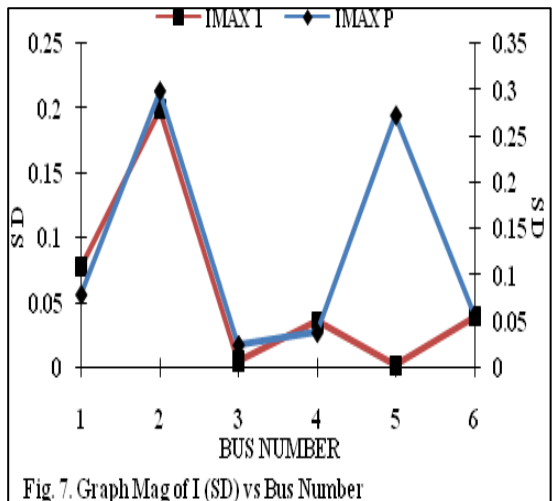
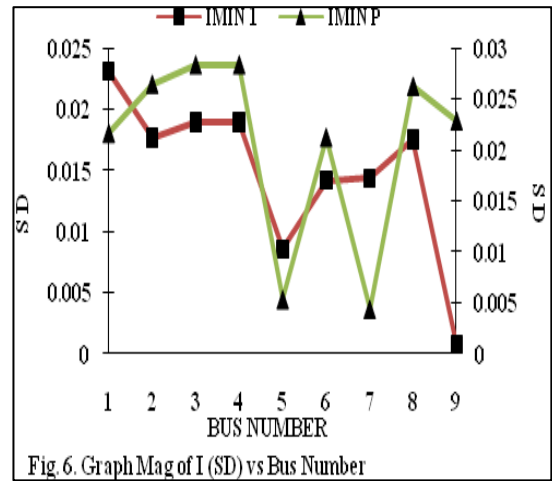
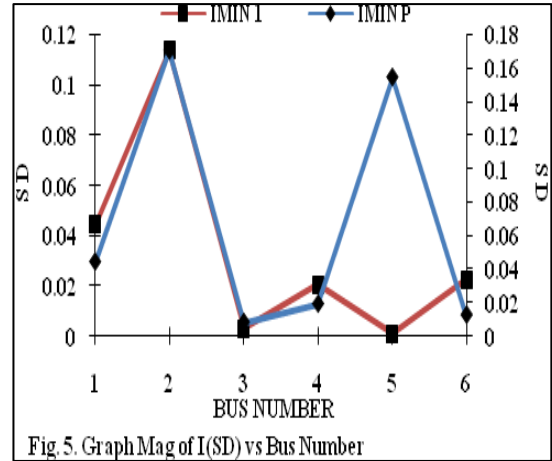


Figure 4 -IEEE 9 Bus System



In this segment, IEEE 6 bus system [5][14] and IEEE 9 bus system [2][7] are tested with their with or without PMU cases to find out the consequences of the PMUs to the precision of the estimated variables.

The parameters measured are current magnitude and real power (flow & injected) measurements. The variation of parameters with or without PMU easily reflected in the fig.5 –12 as below:



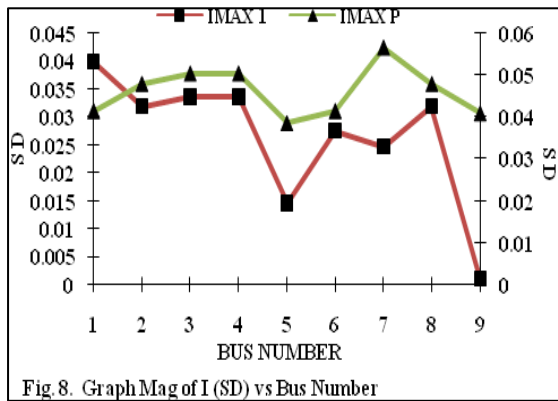


Fig. 8. Graph Mag of I (SD) vs Bus Number

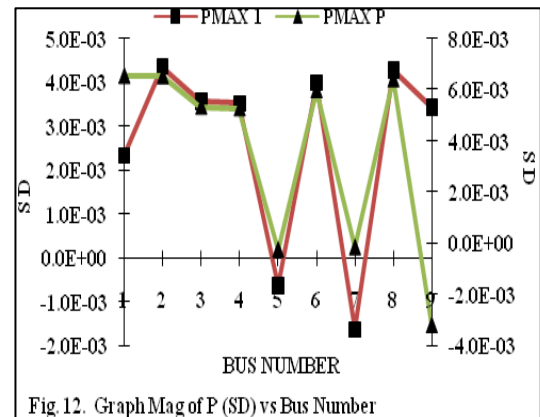


Fig. 12. Graph Mag of P (SD) vs Bus Number

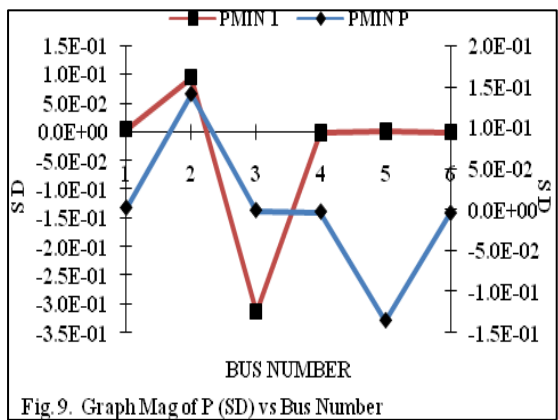


Fig. 9. Graph Mag of P (SD) vs Bus Number

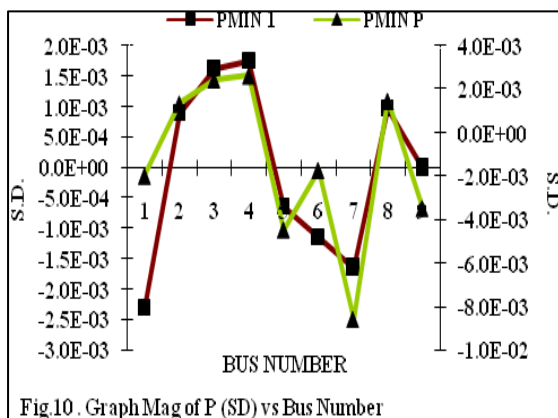


Fig. 10. Graph Mag of P (SD) vs Bus Number

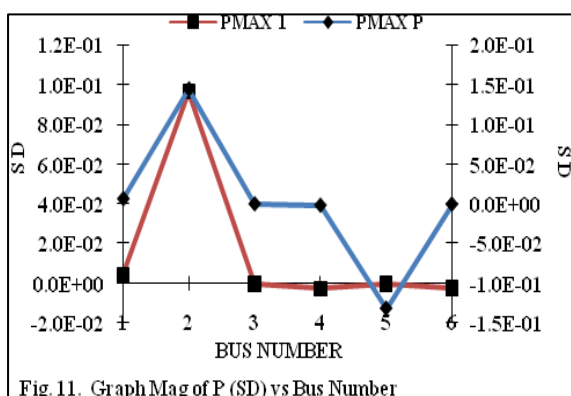


Fig. 11. Graph Mag of P (SD) vs Bus Number

Table 3: Average Std. Dev. of the Estimated Variables

Type of Var.	Type of System	Case 1		Case P	
		Min	Max	Min	Max
Current (I)	6 Bus	0.03414	0.0592	0.0682	0.12733
	9 Bus	0.01483	0.0266	0.0205	0.04625
Power (P)	6 Bus	-0.03661	0.0160	0.0001	0.00265
	9 Bus	-0.00005	0.0025	-0.0014	0.00361

The table 3 shows that how the S.D. values at each case are increases as compared to the S.D. of 'Case P'. In IEEE 9 bus system, the S.D. of the estimated current magnitude is approximately 0.02663 when there is no PMUs, but after adding PMUs to the system, it becomes nearly 0.046256. It means that the S.D. of 'No PMUs' is increased by adding of PMUs. The interesting thing is that the standard deviation increasing as increasing PMU. Therefore, this result shows that the effectively installing of PMUs is reducing the chances of error in measurement of estimated variables.

The Average Current and Average Real Power (flow & injected) are analyze on IEEE 6 Bus & IEEE 9 Bus System (where 6I & 9I for without PMU device and 6P & 9P for with PMU device). The variation of these parameters with or without PMU reflected in the fig.13 –16 as below:

Fig. 13. Current for 6 & 9 Bus System

I MIN61 I MIN91 I MIN6P I MIN9P

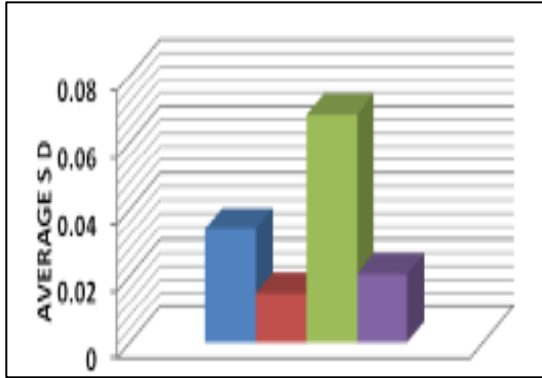


Fig. 15. Real Power for 6 & 9 Bus System

P MIN61 P MIN91 P MIN6P P MIN9P

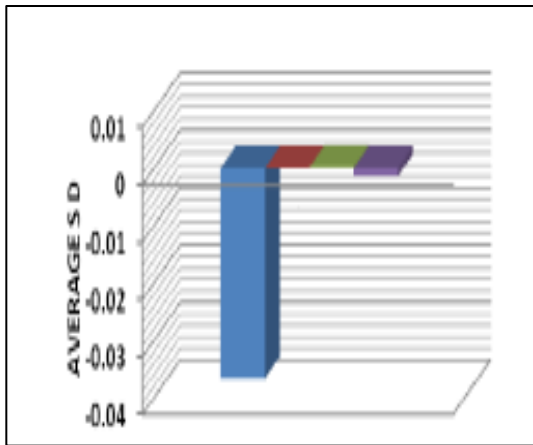


Fig. 14. Current for 6 & 9 Bus System

I MAX61 I MAX91 I MAX6P I MAX9P

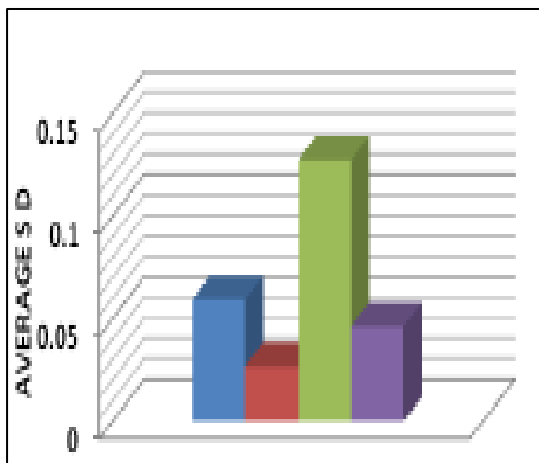
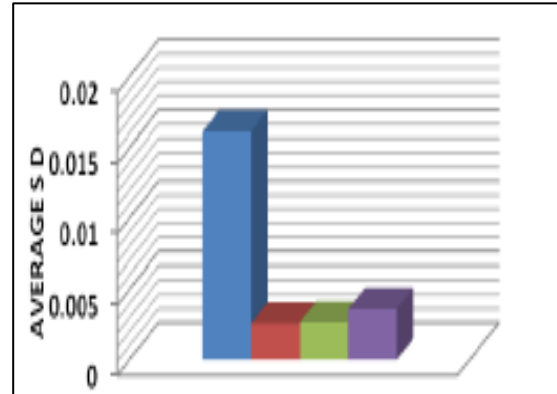


Fig. 16. Real Power for 6 & 9 Bus System

P MAX61 P MAX91 P MAX6P P MAX9P



7.0 Conclusions

In this paper conventional method has been discussed with incorporation of PMU. The study is also carried out to establish the correlation between conventional method and full weighted state estimation with the PMU. Further it is envisage that the result obtained will provide more precise and accurate information of current & power variation on each bus of each system as compared to the conventional method. Such formulation will provide us a more precise and accurate information of current & power variation without doing bulky iteration as in conventional method. By using a single PMU will lead to improved result in respect of accuracy with high resolution.

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