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Multi resolution Analysis of Rainfall Variability Using Discrete Wavelet Transforms

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ABSTRACT

The understanding of the percentage departure in rainfall from average with respect to time is very important for agriculture, environment and economy of any region. The rainfall variability on basis of its average monthly record during the period from Jan. 2013 to Dec. 2017 (05 years) in Moradabad, Uttar Pradesh, India, is discussed. Approximations are low frequency and details are high frequency content of the signal and hence play important role in signal or data analysis. Approximations provide information about the trend of signal. The trend is its slowest part of the signal and represents the behavior of signal corresponding to greatest scale value. A peak in the details shows rapid change or fluctuation in the quantity of rainfall in that time period. Skewness is a measure of symmetry and more precisely, the lack of symmetry while Kurtosis is a measure of whether the data are peakedness or flatness relative to a normal distribution. There is slight significant decreasing trend for long term northeast monsoon rainfall over Moradabad region.

Keywords: *Rainfall; Variability; Wavelet; Approximation; Detail, etc.*

1.0 Introduction

Water is one of the most important natural resources on earth that is necessary for survival of all forms of life. The availability of water for various purposes is very much depending upon the amount of precipitation in that particular region. Rainfall affects the environment and society in various ways ranging from water availability for livelihood, agriculture for functioning of various industries, hydroelectric power generation etc. and thus affects the every aspect of life in that region. In order to understand the hydrological balance and the complex interactions among the components within the hydrologic cycle, we need to focus ourselves on all the information regarding precipitation. Understanding rainfall variability is essential to optimally manage the scarce water resources that are under continuous stress due to the increasing water demands, increase in population and the economic development [1]. Though, climate change is global in nature, potential changes may be consists of some dramatic regional differences. Long term rainfall patterns may get influenced by the global climatic changes and this may result with the danger of increasing the occurrences of droughts and floods. Consequently, the water management using all the available resources is becoming more challenging and crucial day by day. Therefore, the study and understanding of variation in rainfall in time and its attendant effects on the ecosystem is very important [2].

Moradabad (UP, India) city is very near to the forest and hill range of Uttarakhand (about 60 Km). This forest and hill range direct affects the weather and climate of Moradabad region. It is an agriculture-based district where more than 80% of its people are directly or indirectly engaged in a wide range of agricultural activities. The rainfall is changing on both the global and the regional scale due to global warming, which is well established. Study on rainfall

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variability and their statistical analysis are therefore important for long-term water resources planning, agricultural development and disaster management mainly in the context of global climatic change.

The discrete nature of rainfall in time and space has always posed unique problems for the climatologist compared to more continuous climatic elements such as temperature and pressure.

Wavelets are introduced as basis functions by which time resolution must increase with the central frequency of the analysis filters.

Of course they still satisfy the Heisenberg inequality, but now, the time resolution becomes arbitrarily good at high frequency, while the frequency resolution becomes arbitrarily good at low frequencies [3].

For practical applications, and for efficiency reasons, one prefers continuously differentiable functions with compact support as mother (prototype) wavelet (functions).

However, to satisfy analytical requirements (in the continuous WT) and in general for theoretical reasons, one chooses the wavelet functions from a subspace of the space.

2.0 Basics of Wavelet Transforms

A Physicist and an Engineer, Grossmann and Morlet, provided a definition and a way of thinking based on physical intuitions that was flexible enough to cover all these cases [4].

According to them, a wavelet is a function ψ in $L^2(\mathbb{R})$, whose Fourier tramsform $\hat{\psi}(\xi)$ satisfies the condition equation 1,

$$\int_{-\infty}^{\infty} \left|\hat{\psi}(\xi t)\right|^2 \frac{dt}{t} = 1 \qquad (1)$$

A wavelet ψ is simply a function of time t that obeys a basic rule, known as the wavelet admissibility condition equation 2,

$$C_\psi = \int_0^{\infty} \frac{|\hat{\psi}(\xi)|}{\xi} d\xi < \infty \qquad (2)$$

where $\hat{\psi}(\xi)$ is the Fourier transform. This condition ensures that $\hat{\psi}(\xi)$ goes to zero quickly as $\xi \to 0$. In fact, to guarantee that $C_\psi < \infty$, we must impose $\hat{\psi}(0) = 0$, which is equivalent to the condition of zero mean equation 3,

$$\int_{-\infty}^{\infty} \psi(t) dx = 0 \qquad (3)$$

And the second condition imposed on wavelet function is unit energy or the condition for square norm one, equation 4:

$$\int_{-\infty}^{\infty} |\psi(t)|^2 dt = 1 \qquad (4)$$

For ψ to be a wavelet for the continuous wavelet transform the mother wavelet must satisfy an admissibility criterion in order to get a stably invertible transform. A sampling of the continuous wavelet transform is equation 5.

$$W_{s,\tau} = \int f(t) \frac{1}{\sqrt{s}} \psi\left(\frac{t-\tau}{s}\right) dt = \int f(t)\, \psi_{s,\tau}(t)\, dt \qquad (5)$$

Where, s is positive and defines the scale and τ is any real number and defines the shift. The pair $(s,)$ defines a point in the right half plane, $\mathbb{R}+\times\mathbb{R}$**.** The projection of a function (t) onto the subspace of scale s then has the form;

$$f_s(t) = \int_{\mathbb{R}} W_\psi f(s,\tau)\, \psi_{s,\tau}(t)\, d\tau$$

with wavelet coefficients given by equation 6:

$$c(s,\tau) = W_\psi f(s,\tau) = \langle f,\, \psi_{s,\tau}\rangle = \int_{\mathbb{R}} f(t)\, \psi^*{}_{s,\tau}(t)\ dt \qquad (6)$$

For some real parameters $s > 1, \tau > 0$, the corresponding discrete subset of the half plane consists of all the points with $j,k \in \mathbb{Z}$**,** The corresponding child wavelets are now given as,

$$\psi_{j,k}(t) = \frac{1}{\sqrt{s^{-j}}} \psi\left(\frac{t - ks^{-j}}{s^{-j}}\right)$$

Discrete wavelet coefficient given by equation 7,

$$c(j,k) = \langle f,\, \psi_{j,k}\rangle = \int_{\mathbb{R}} f(t)\, \psi^*_{j,k}(t)\, dt, \qquad \ldots (7)$$

where $\psi_{j,k}(t) = 2^{j/2}\psi(2^j t - k)$

Inverse discrete wavelet transform,

$$f(t) = \sum_{j\in\mathbb{Z}} \sum_{k\in\mathbb{Z}} c(j,k)\ \psi_{j,k}(t)$$

A sufficient condition for the reconstruction of any signal f of finite energy by the formula,

$$f(t) = \sum_{j\in\mathbb{Z}} \sum_{k\in\mathbb{Z}} \langle f,\, \psi_{j,k}\rangle\ \psi_{j,k}(t)$$

is that the functions $\{\psi_{j,k}: j,k \in \mathbb{Z}\}$ form an orthonormal basis of $L^2(\mathbb{R})$. where j and k are integers representing the set of discrete translations and discrete dilations.

For a choice s=2 and τ=1, we can write discrete wavelet transforms given by equation 8,

$$W_{j,k}f = \int f(t) 2^{j/2} \psi(2^j t - k)\, dt \qquad (8)$$

We call $\psi_{j,k}(t) = \psi(t)$ as mother wavelet. Other wavelets are produced by translation and dilation of the mother wavelet.

The purpose of the study is to explore the trend of departure of rainfall from average in the Moradabad region. The rainfall variability during the period from Jan. 2013 to Dec. 2017 (5 years), in Moradabad, Uttar Pradesh, India, is analyzed. Most of the rainfall is found out to be taking place in Northeast monsoon season. The analysis revealed great degree of variability in precipitation with time. Information about the trends of rainfall are important as it is closely related to the practical water relates issues in the region especially flood related problems. Thus it becomes increasingly important to study the trend in rainfall variability and its physical explanation.

3.0 Data Methodology

In wavelet applications, mostly we do not deal with scaling functions and associated wavelets, we need only □□ and □□ in the refinement relation and the signal [5, 6]. The □□ and □□ are viewed as filters and the sequence of data as digital signal. Most of the results in wavelet theory are explained in terms of signal expansion or in terms of filter banks. The scaling functions and associated wavelets do not play role directly in the computation of signal expansion coefficients. This is because of the relationship between expansion coefficients at a lower scale and expansion coefficients at a higher scale. We use Haar scaling and wavelet functions to establish the relation between these coefficients. We have the refinement relation responsible for signal analysis is given by equation 9:-

$$\phi(t) = \sum_{k=0}^{N-1} \alpha_k \sqrt{2}\, \phi(2t-k) = \sum_{k=0}^{N-1} \alpha_k\, \phi_{1,k}(t) \qquad (9)$$

where **N** is the number of coefficients in the refinement relation. Here **αk** and **βk** are also called low pass scaling filter and high pass wavelet filter coefficients. The space is given by equation 10 and 11.

$$\mathbf{V_j} = \underset{\mathbf{k}}{\mathbf{span}}\ \overline{\{\phi 2^j t - k\}} \qquad (10)$$

$$\mathbf{W_j} = \underset{\mathbf{k}}{\mathbf{span}}\ \overline{\{\psi 2^j t - k\}} \qquad (11)$$

from orthogonality relation $\mathbf{V_{j+1}} = \mathbf{V_j} \oplus \mathbf{W_j}$ follow the existence of sequences $\boldsymbol{\alpha} = \{\alpha_k\}_{k\in Z}$ and $\boldsymbol{\beta} = \{\beta_k\}_{k\in Z}$ that satisfy the identities is given by equation 12 and 13,

$$\alpha_k = \langle \phi_{0,0}, \phi_{1,k} \rangle \text{ so that } \phi(t) = \sqrt{2} \sum_{k\in Z} \alpha_k\, \phi(2t-k) \qquad (12)$$

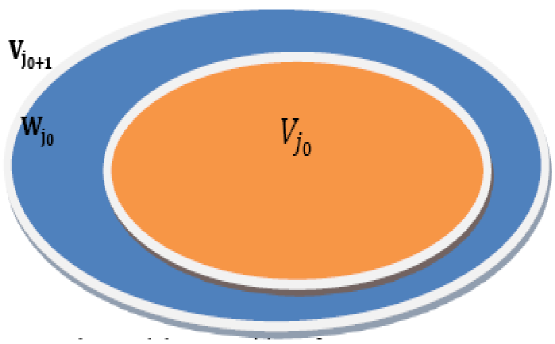
and

$$\beta_k = \langle \psi_{0,0}, \phi_{1,k} \rangle \text{ so that } \psi(t) = \sqrt{2} \sum_{k\in Z} \beta_k\, \phi(2t-k) \qquad (13)$$

From the multiresolution analysis, the orthogonal decomposition of space $\mathbf{L^2}(\mathbb{R})$ [7];

$$\mathbf{L^2}(\mathbb{R}) = \sum_{j} \mathbf{V_j} = \mathbf{V_{j_0}} \oplus \mathbf{W_{j_0}} \oplus \mathbf{W_{j_0-1}} \oplus \mathbf{W_{j_0-2}} \oplus \mathbf{W_{j_0-3}} \oplus \mathbf{W_{j_0-4}} \ldots\ldots\ldots$$

Fig 1: Orthogonal Decomposition of Space



For any signal or function $S \in L^2(\mathbb{R})$, this gives a representation in basis functions of the corresponding subspaces as;

$$\mathbf{S} = \sum_{k} \mathbf{c_{j_0,k}}\, \boldsymbol{\phi}_{\mathbf{j_0,k}} + \sum_{j\le j_0}^{1} \sum_{k} \mathbf{d_{j,k}}\, \boldsymbol{\psi}_{\mathbf{j,k}}$$

where the coefficients are;

$$\mathbf{c_{j_0,k}} = \langle \mathbf{S}, \boldsymbol{\phi}_{\mathbf{j_0,k}} \rangle$$

$$\mathbf{d_{j,k}} = \langle \mathbf{S}, \boldsymbol{\psi}_{\mathbf{j,k}} \rangle$$

For Haar N = 2 and the relation is;

$\phi(t) = \alpha_0 \sqrt{2}\, \phi(2t) + \alpha_1 \sqrt{2}\, \phi(2t-1) = \alpha_0\, \phi_{1,0}(t) + \alpha_1\, \phi_{1,1}(t)$ (3.6)

and $\psi(t) = \beta_0\, \phi_{1,0}(t) + \beta_1\, \phi_{1,1}(t)$

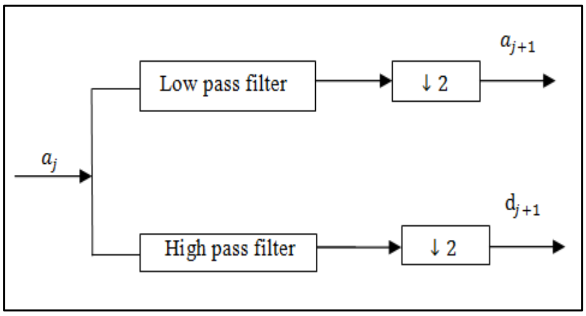
A simple trick to analyze the signal is to rewrite a_0 as a telescope sum [8],

$$a_0 = a_j + (a_{j-1} - a_j) + (a_{j-2} - a_{j-1}) + \cdots \ldots\ldots\ldots + (a_1 - a_2) + (a_0 - a_1) = a_j + d_j + d_{j-1} + \ldots\ldots\ldots\ldots\ldots\ldots d_2 + d_1$$

The important difference is that for each new j, we will now only send the additional details needed for computing a_{j+1} from a_j:

$$a_j = a_{j+1} + d_{j+1} \quad \text{where } d_{j+1} = a_j - a_{j+1}$$

Fig 2: Decomposition of wavelet coefficients



For d1 we can repeat exactly the same kind of computation on each of the intervals [0,1/2] and [1/2,1] to get is given by equation 14

$$d_1(t) = a_0(t) - a_1(t) = \beta_0\psi\,(2t) + \beta_1\psi\,(2t-1) \qquad (14)$$

with $\beta_k = \int_{\mathbb{R}} f(t)\, 2\, \psi\,(2t-k)dt$. For larger j, it follows in exactly the same way for $d_j(t)$ that is given by equation 15 and 16

$$d_j(t) = a_{j-1}(t) - a_j(t) = \sum_{k=0}^{2^j-1} \beta_k \psi\,(2^j t - k) \qquad (15)$$

with

$$\beta_k = \int_{\mathbb{R}} f(t)\, 2^j \psi\,(2^j t - k)dt \qquad (16)$$

So far we have restricted to the interval [0,1], but the same procedure can be repeated in each integer endpoint interval [k,k + 1]. A function f(t) has a wavelet series expansion is given by equation 17;
f(t)=Σcj0,k ϕk∈ℤ(t−k)+ΣΣdj,k(t)ψj,k(t)k∈ℤ∞j=0 (17)

An arbitrarily detailed approximation can be computed by truncations to finite sums. It also follows that the sum Σcj0,k ϕ(t−k)k∈ℤ is the orthogonal projection of f on the space Vj0 of square integrable functions that are constant on integer endpoint intervals [k,k + 1). For j = 1, the sum Σdj,kψj,kk∈ℤ(t) adds the details necessary to get an approximation in the space Vj0+1 of square integrable functions that are constant on all intervals. Multi-resolution analysis uses wavelets as a tool for describing the increment of information needed to go from a coarse approximation to a higher resolution approximation.

4.0 Results and discussion

The rainfall variability data from Jan. 2013 to Dec. 2017 is taken as original signal S which represents the average monthly behavior of the percentage departure of rainfall from average during this period shown by figure 3 and figure 4.

Fig 3: Percentage Departure of Rainfall from 01/01/2013 to 31/12/2017 in Moradabad

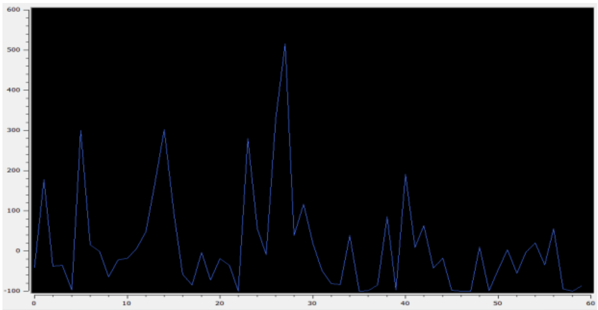
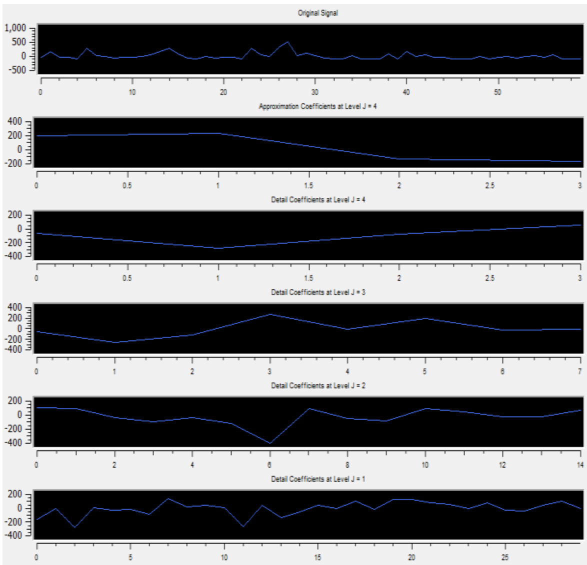


Fig 4: Wavelet Decomposition of Percentage Departure of Rainfall



Approximations a_1, a_2, a_3, a_4 represent 2^1, 2^2, 2^3, 2^4 months average behavior of the signal. Likewise details represent 2^1, 2^2, 2^3, 2^4 months variations in the signal. In wavelet analysis, Approximations are low frequency and details are high frequency content of the signal. Due to low frequency content, the approximations are very important part of the signal and provide information about the trend of signal. The trend represents the behavior of signal corresponding to greatest scale value. In Fig. 4 the approximations a_4 and details d_1, d_2, d_3, d_4 belonging to Haar wavelet decomposition up to level 4 are shown. As the scale value is increased, the resolution of the signal decreases, that provides better estimation of unknown trend of the signal. From the trend of the signal, it is obvious that the average rainfall is continuously decreasing in the Moradabad region in that time period. The decrement of average rainfall is mainly due to anthropogenic driving forces. A peak in the details shows rapid change or fluctuation in the quantity of rainfall in that time mode.

Table 1: Statistical Parameters of Rainfall Variability

S. No.	Parameter	Value
1	Skewness	1.909011
2	Kurtosis	4.067868
3	Standard Deviation	126.1731

Skewness is a measure of lack of symmetry while Kurtosis is a measure of whether the data are flatness relative to a normal distribution [9].

5.0 Conclusions

With the help of discrete wavelet transforms using Haar wavelet the 05 years data based upon average monthly record is decomposed up to level 04. The trend of the rainfall variability becomes more sharp and clear from the plot of approximation coefficients of level 4. From the trend of the signal, it is obvious that the average rainfall is decreasing in the Moradabad region in that time period. A peak in the details shows rapid change or fluctuation in the quantity of rainfall in that time period. Positive value of skewness indicates that the rainfall data is skewed to right. High positive value of kurtosis indicates the strong intermittency in the rainfall variability. High value of standard deviation indicates that the data points are spread out over a wider range of values. Approximation and details coefficients are drawn in Fig. 4, using Haar wavelet transforms. Analyzing the time series it is clear that the energy of high pass filtered time series has a small contribution to the time series as compared with the low-pass filtered time series.

Taking into account these results we have shown that the generalized wavelet analytical approach provides a simple and accurate framework for modeling the statistical behavior of rainfall variability data involved in the climate dynamics. From the present analyses, we found that the departure of rainfall time-series in last 05 years in Moradabad is highly intermittent.

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