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Applicability of Optimization Principles for Valve Sourcing and Expediting –The Indian Valve Sourcing and Expediting Series

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ABSTRACT

Sourcing involves alot of transportation activities and it is the role of the sourcing office to minimize the transportation cost. With such an objective, it is the necessity of the hour to compare and optimize the transportation costs from various sources to destinations. Optimization is a vast technique, which when applied diligently will have very serious benefits such as quality improvement and efficiency improvement in any process. By converting the real time transportation case into a Linear Programming problem of optimization, costs can be tremendously brought down. This article has discussed about the various techniques to minimize the transportation costs. Major methods such as North-West Corner Method, Least Cost Entry method, MODI method have been discussed. The sourcing office teams often use Microsoft Excel for their day-to-day sourcing activities. Hence an algorithm to apply these techniques with the aid of Microsoft Excel will be released in the upcoming episodes of the series.

Keywords: *Linear programming; Optimization; Valve Sourcing.*

1.0 Introduction

Sourcing in India is blooming at a faster rate, the activities being dominated by many small scale industries across India. Also, it is very easy to source a product from India as India has a substantial quantity of small and middle scale industries. Valve sourcing is an upcoming activity in India and the numbers have increased tremendously over the past decade. Valve sourcing from India, either requires the Parent Organization (PO) to set up its own office in India or requires the help of a Sourcing Agent (Third Party) who will perform sourcing and expediting activities by setting up a team of experts. The major activities in Valve Sourcing are:

1. Appraisals to select suitable industries (suppliers) to manufacture valves for the required specification.
2. Establishment of a contract between the PO and the supplier – Technical and Financial.
3. Expediting Visits to accelerate the manufacturing process, such that the product is delivered within the specified time.
4. Quality Inspections to report to the PO about the technical quality of the valves.
5. Monitoring the Logistics activities such that the products reach the parent organization on time.

As a sourcing manager operating in India, the person must also consider the transportation costs that will be often paid by the PO. If there are a number of qualifying suppliers, then a comparative allocation shall be made based on the transportation cost. Hence optimization techniques have to be applied to minimize the transportation costs. Major categories of optimization include Linear Programming, Non-Linear Programming, Dynamic Programming and Bionic Inspired Optimization.

Transportation problem is a class of Linear Programming problem [1] that can be adopted to estimate the minimum transportation cost when there are multiple sources and multiple destinations for the same product. Several researchers have investigated on several different algorithms [2-5] to obtain a feasible solution for the case. The problem can be either of balanced type, where the number of sources and destinations are equal or it can be of

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unbalanced type, where sources and destination are unequal.

Out of various available algorithms to calculate the initial basic feasible solution, the most prominent are:

1. North-West Corner Method [6,7]
2. Least Cost Entry Method [8]
3. Modified Distribution Method (MODI) [9,10]

Other than these prominent methods, few other methods such as Vogel's Approximation Method, Maximum Minimum Total Opportunity Cost Method and ICMM Method are also available and are applicable to certain specified cases.

This research work focuses in applying transportation technique to sourcing and expediting computations to maximize the benefits and to minimize the cost of transportation.

2.0 Algorithm of Transportation Problem

Consider an example of a real time implementation. A valve manufacturing company in the United States of America sources 12" Resilient Wedge Gate Valve from India, China and from UAE and has to supply to 2 Assembly Units in USA and 2 in Canada.

In such a case, the Global Sourcing Head/ Global Sourcing Manager will have to take into consideration the transportation charges that will be incurred from various sources.

In such a case, the Global Sourcing Manager will have to make a comparison between various available sources and will have to allocate appropriate quantity from each source to destination that will minimize the total cost of transportation.

This transportation method of Linear Programming can be well extended to objectives such as,

1. Minimization of Transportation Cost
2. Minimization of Delivery Time
3. Maximization of Profits

Consider the previous example, for the company in USA, there are three sources and four destinations. The case can be pictorially represented as in Fig.1

In such a case, equating the source supply

$q_{11} + q_{12}+q_{13}+q_{14} = a_1$ (1)

$q_{21} + q_{22}+q_{23}+q_{24} = a_2$ (2)

$q_{31} + q_{32}+q_{33}+q_{34} = a_3$ (3)

Equating the demands at destinations,

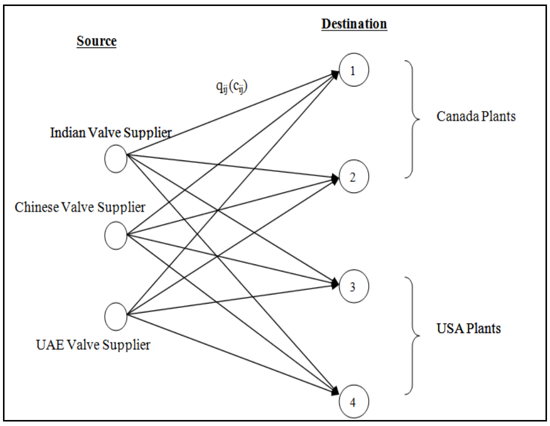
$q_{11} + q_{21}+q_{31}= b_1$ (4)

$q_{12} + q_{22}+q_{32}= b_2$ (5)

$q_{13} + q_{23}+q_{33}= b_3$ (6)

$q_{14} + q_{24}+q_{34}= b_4$ (7)

Fig 1: Transportation Problem Algorithm



Where, 'q_{ij}' is the quantity transported from source 'i' to destination 'j'

'c_{ij}' is the cost of transportation of unit product from source 'i' to destination 'j'

Assume the following terminologies for parameters as shown in Table 1

Table 1: Terminologies Assumed for the Algorithm

a_1	Total supply from Source 1
a_2	Total supply from Source 2
a_3	Total supply from Source 3
b_1	Total demand at destination 1
b_2	Total demand at destination 2
b_3	Total demand at destination 3
b_4	Total demand at destination 4

Eq(1-7) explain the technical scenario mathematically. This can be represented in tabular form as shown in Table 2. The cells in the table represent the quantity to be transported from each source to the destination along with the unit cost of transportation that will be incurred for the case.

Table 2: Mathematical form of the Real Time Sourcing Case

	Destination				Supply
	1	2	3	4	
From Indian Valve Supplier (Source 1)	q_{11} (c_{11})	q_{12} (c_{12})	q_{13} (c_{13})	q_{14} (c_{14})	a_1
From Chinese Valve Supplier (Source 2)	q_{21} (c_{21})	q_{22} (c_{22})	q_{23} (c_{23})	q_{24} (c_{24})	a_2
From UAE Supplier (Source 3)	q_{31} (c_{31})	q_{32} (c_{32})	q_{33} (c_{33})	q_{34} (c_{34})	a_3
Demand	b_1	b_2	b_3	b_4	

Hence, the total cost of transportation can be represented as,

$$C = \sum_{i=1}^{n} \sum_{j=1}^{m} q_{ij} c_{ij}$$

where,n – no. of sources,m – no. of destinations.

The main objective of the case is to minimize the transportation cost. Hence the Linear Programming Problem can be represented with the following objective and constraints,

Objective:	Minimize $C = \sum_{i=1}^{n} \sum_{j=1}^{m} q_{ij} c_{ij}$
Constraints:	$\sum_{j=1}^{m} q_{ij} = a_i$, i= 1,2,3…n $\sum_{i=1}^{n} q_{ij} = b_j$, j= 1,2,3…m $q_{ij} \geq 0$ for all i,j

The following steps have to be followed to calculate the minimum cost of transportation,

Step 01:	Convert the real time case into mathematical form
Step 02:	Determine the Initial Basic Feasible Solution (BFS)
Step 03:	Perform an optimality test
Step 04:	If optimal solution is reached, calculate the cost of transportation
Step 05:	If optimal solution is not reached, perform iterations to compute the optimal Initial BFS

3.0 Calculation of Minimum Transportation Cost using North-West Corner Method

In this algorithm, the quantity to be supplied is spread out in various cells of a table such that their total remains the same. The mathematical procedure to calculate the transportation cost using this method can be explained with the example.
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3.1 Example-1

A Company X, which is having its headquarters at Norway is planning to source check valves of a certain specification from a supplier at UAE, a supplier at Indonesia and a supplier at India. It has to transport the sourced valves to its four assembly units, two at Norway and the other two at Canada. The cost that will be incurred for the transportation has been tabulated (Cost in INR). Minimize the transportation cost and obtain a suitable allocation plan for sourcing valves.

Table 3: Transportation Cost and Supply Demand Data

	Destination				**Supply Capacity**
	Norway Units		**Canada Units**		
	1	**2**	**3**	**4**	
	(INR)				**(Qty)**
From UAE Valve Supplier (Source 1)	20	30	50	10	**80**
From Indonesia Valve Supplier (Source 2)	70	30	40	60	**100**

From Indian Supplier (Source 3)	40	10	70	20	**200**
Demand (Qty)	**60**	**80**	**90**	**150**	**380**

3.2 Solution to example-1

Here in this case, Supply = Demand = 380 Units. In this method, the north-west corner cell is first operated and the corresponding supply and demand corresponding to that particular cell is compared and whichever is lower is allocated.

If in the case, demand is nullified, the entire corresponding row has to be eliminated and if supply is nullified, the entire column will be eliminated.

The following tables will explain the process of computing the BFS.

Table 4: Calculation of Initial BFS

	Destination				
	~~**1**~~	**2**	**3**	**4**	**S**
1	~~**60**~~ ~~$_{20}$~~	30	50	10	**80-60 =20**
2	~~70~~	30	40	60	**100**
3	~~40~~	10	70	20	**200**
D	**60 - 60 = 0**	**80**	**90**	**150**	

In this case 60 < 80 and hence , 60 units were allocated and by the allocation, Column 1 got nullified and hence was cancelled.

In the next step, the next North-West Corner Cell is operated. As the rows and columns get nullified, the subsequent northwest corner cell is operated.

Table 5: Calculation of Initial BFS

	Destination				
	~~**1**~~	**2**	**3**	**4**	**S**
~~1~~	~~**60**~~ ~~$_{20}$~~	~~**20**~~ ~~$_{30}$~~	~~50~~	~~10~~	**80-60 =20 20-20 = 0**
2	~~70~~	30	40	60	**100**
3	~~40~~	10	70	20	**200**
D	**60 - 60 = 0**	**80 -20 = 60**	**90**	**150**	

Table 6. Calculation of Initial BFS

	Destination				
	~~**1**~~	~~**2**~~	~~**3**~~	**4**	**S**
~~1~~	~~60~~ ~~$_{20}$~~	~~20~~ ~~$_{30}$~~	~~50~~	~~10~~	**80-60 =20 20-20 = 0**
~~2~~	~~70~~	~~60~~ ~~$_{30}$~~	~~40~~ ~~$_{40}$~~	~~60~~	~~**100 – 60 = 40 40 – 40 = 0**~~
3	~~40~~	~~10~~	~~50~~ ~~$_{70}$~~	20	**200 – 50 = 150**
D	**60 - 60 = 0**	**80 -20 = 60 60 – 60 = 0**	**90 – 40 = 50 50 – 50 = 0**	**150**	

Table 7: Calculation of Initial BFS

	Destination				
	~~**1**~~	~~**2**~~	~~**3**~~	**4**	**S**
~~1~~	~~60~~ ~~$_{20}$~~	~~20~~ ~~$_{30}$~~	~~50~~	~~10~~	**80-60 =20 20-20 = 0**
~~2~~	~~70~~	~~60~~ ~~$_{30}$~~	~~40~~ ~~$_{40}$~~	~~60~~	~~**100 – 60 = 40 40 – 40 = 0**~~
~~3~~	~~40~~	~~10~~	~~50~~ ~~$_{70}$~~	~~150~~ ~~$_{20}$~~	**200 – 50 =**

					150 150 – 150 = 0
D	**60 - 60 = 0**	**80 -20 = 60 60 – 60 = 0**	**90 – 40 = 50 50 – 50 = 0**	**150 – 150**	

Hence the transportation cost will be = [(60 × 20) + (20 × 30) + (60 × 30) + (40 × 40) + (50 × 70) + (150 × 20)] =Rs. 11,700

This solution must be checked for optimality. The optimality test consists of two phases. Non-degeneracy test and Cell Evaluation Test Only when the found solution passes both the tests, it will be considered as an optimal solution.

3.3 Non degeneracy test

For the solution to be non-degenerate, it must obey the following equation,

Individual Allocations= (No.of sources) + (No. of destinations) - 1

In this case,Individual allocations = 6, Number of sources = 3

Number of destinations = 4

Hence, 4+3-1 = 6. Hence the condition is satisfied

3.4 Cell evaluation test.

In this test, the cost coefficients of the allotted cells are split into x_i and y_j components,

Table 8: Calculation of Cost Components

	Destination				
	1	**2**	**3**	**4**	**x_i**
1	20	30			**0**
2		30	40		**0**
3			70	20	**30**
y_j	**20**	**30**	**40**	**-10**	

The x_i and $y_j$components for each allocated cell are calculated as follows. Initially assume x_1 = 0

$c_{11} = x_1 + y_1 = 20 \rightarrow y_1 = 20$

$c_{12} = x_1 + y_2 = 30 \rightarrow y_2 = 30$

$c_{22} = x_2 + y_2 = 30 \rightarrow x_2 = 0$

$c_{23} = x_2 + y_3 = 40 \rightarrow y_3 = 40$

$c_{33} = x_3 + y_3 = 70 \rightarrow x_3 = 30$

$c_{34} = x_3 + y_4 = 20 \rightarrow y_4 = -10$

For the solution to be optimal,

$$\mathbf{[m_{ij}] \geq 0 \text{ where}, [m_{ij}] = [c_{ij}] - [x_i + y_j]}$$

Where, $\mathbf{[c_{ij}]}$ **=** cost matrix of unallocated cells

In this case,

$$[c_{ij}] = \begin{bmatrix} & & 50 & 10 \\ 70 & & & 60 \\ 40 & 10 & & \end{bmatrix} \& [x_i + y_j] = \begin{bmatrix} & & 40 & -10 \\ 20 & & & -10 \\ 50 & 60 & & \end{bmatrix}$$

$$\mathbf{[m_{ij}] = } = \begin{bmatrix} & & 10 & 20 \\ 50 & & & 70 \\ -10 & -50 & & \end{bmatrix} \mathbf{< 0}$$

The achieved solution is not the optimal one. Hence the BFS computed by North – West corner method is not valid and has to be improved. The iterative method to improve the solution will be dealt in the upcoming sections.

3.5 Example 2

The Global Sourcing Manager of an Indian Valve Manufacturing company is planning to source Triple offset butterfly valves of certain specification from suppliers at Belgium, Germany and USA and is planning to supply to its three manufacturing units at Coimbatore, Pune and Aizawl. The cost of transportation is tabulated. Estimate the minimum cost of transportation and suggest suitable allocation.

Table 9: Transportation cost and Supply Demand Data

	Destination			**Supply Capacity**
	1 Coimbatore	**2 Pune**	**3 Aizawl**	
	(INR)			**(Qty)**
Belgium (Source 1)	320	600	2000	**200**

Germany (Source 2)	400	680	800	**300**
USA (Source 3)	1200	1040	600	**450**
Demand (Qty)	**300**	**350**	**300**	**950**

3.6 Solution for example-2

Here in this case Supply = Demand = 950 Units. The following table explains the application of N-W Corner method to the case.

Table 10: Calculation of Initial BFS

	Destination			
	1	**2**	**3**	**S**
~~1~~	~~200~~ ~~$_{320}$~~	~~$_{600}$~~	~~$_{2000}$~~	**200 – 200 = 0**
2	$_{400}$	$_{680}$	$_{800}$	**300**
3	$_{1200}$	$_{1040}$	$_{600}$	**450**
D	**300 – 200 = 100**	**350**	**300**	

Table 11: Calculation of Initial BFS

	Destination			
	~~**1**~~	**2**	**3**	**S**
~~1~~	~~200~~ ~~$_{320}$~~	~~$_{600}$~~	~~$_{2000}$~~	**200 – 200 = 0**
2	~~100~~ ~~$_{400}$~~	$_{680}$	$_{800}$	**300 – 100 = 200**
3	~~$_{1200}$~~	$_{1040}$	$_{600}$	**450**
D	**300 – 200 = 100** **100 – 100 = 0**	**350**	**300**	

Table 12: Calculation of Initial BFS

	Destination			
	~~**1**~~	**2**	**3**	**S**
~~1~~	~~200~~ ~~$_{320}$~~	~~$_{600}$~~	~~$_{2000}$~~	**200 – 200 = 0**
~~2~~	~~100~~ ~~$_{400}$~~	~~200~~ ~~$_{680}$~~	~~$_{800}$~~	**300 – 100 = 200** **200 – 200 = 0**
3	~~$_{1200}$~~	$_{1040}$	$_{600}$	**450**
D	**300 – 200 = 100** **100 – 100 = 0**	**350 – 200 = 150**	**300**	**950**

Table 13: Calculation of Initial BFS

	Destination			
	~~**1**~~	~~**2**~~	**3**	**S**
~~1~~	~~200~~ ~~$_{320}$~~	~~$_{600}$~~	~~$_{2000}$~~	~~**200 – 200 = 0**~~
~~2~~	~~100~~ ~~$_{400}$~~	~~200~~ ~~$_{680}$~~	~~$_{800}$~~	~~**300 – 100 = 200**~~ ~~**200 – 200 = 0**~~
3	~~$_{1200}$~~	~~150~~ ~~$_{1040}$~~	$_{600}$	**450 –**

				150 = 300
D	**300 – 200 = 100** **100 – 100 = 0**	**350 – 200 = 150** **150 – 150 = 0**	**300**	

Table 14. Calculation of Initial BFS

	Destination			**S**
	~~**1**~~	~~**2**~~	**3**	
~~1~~	~~200_{320}~~	~~600~~	~~2000~~	**200 – 200 = 0**
~~2~~	~~100_{400}~~	~~200_{680}~~	~~800~~	**300 – 100 = 200** **200 – 200 = 0**
3	~~1200~~	~~150_{1040}~~	300_{600}	**450 – 150 = 300** **300 – 30**

				0 = 0
D	**300 – 200 = 100** **100 – 100 = 0**	**350 – 200 = 150** **150 – 150 = 0**	**300 – 300 = 0**	

Total Cost of Transportation = [(320 × 200) + (400 × 100) + (680× 200) + (1040 × 150) + (600 × 300)] = Rs. 5,76,000
The optimality of this solution has to be checked.

3.7 Non Degeneracy Test

Individual Allocations= (No.of sources) + (No. of destinations) - 1
In this case,Individual allocations = 5, Number of sources = 3
Number of destinations = 3
Hence, 3+3-1 = 5 . Hence the condition is satisfied

3.8 Cell Evaluation Test

Table 15. Calculation of Cost Components

	Destination			x_i
	1	**2**	**3**	
1	320			**0**
2	400	680		**80**
3		1040	600	**160**
y_j	**320**	**600**	**440**	

The x_i and $y_j$components for each allocated cell are calculated as follows. Initially assume x_1 = 0

$c_{11} = x_1 + y_1 = 320 \rightarrow y_1 = 320$
$c_{21} = x_2 + y_1 = 400 \rightarrow x_2 = 80$
$c_{22} = x_2 + y_2 = 680 \rightarrow y_2 = 600$
$c_{32} = x_3 + y_2 = 1040 \rightarrow y_3 = 440$
$c_{33} = x_3 + y_3 = 600 \rightarrow x_3 = 160$

For the solution to be optimal,

$$[m_{ij}] \geq 0 \text{ where, } [m_{ij}] = [c_{ij}] - [x_i + y_j]$$

Where, **$[c_{ij}]$ =** cost matrix of unallocated cells

In this case,

$$[c_{ij}] = \begin{bmatrix} & 600 & 2000 \\ & & 800 \\ 1200 & & \end{bmatrix} \& \ [x_i + y_j] = \begin{bmatrix} & 600 & 440 \\ & & 520 \\ 480 & & \end{bmatrix}$$

$$[m_{ij}] = = \begin{bmatrix} & 0 & 1560 \\ & & 280 \\ 720 & & \end{bmatrix} \geq 0$$

Hence the optimal solution is achieved.

Suggested allocation

1. Belgium supplier to Coimbatore Plant : 200 units
2. Germany supplier to Coimbatore Plant: 100 units
3. Germany supplier to Pune Plant: 200 units
4. USA supplier to Pune Plant: 150 units
5. USA supplier to Aizawl Plant: 300 Units

Minimum Transportation Cost that will be achieved through this allocation = Rs. 5, 76,000 /-

4.0 Calculation of Minimum Transportation Cost Using Lowest Cost Entry Method

This method is similar in operation to NW Corner method, except for that the entries are nullified based on the minimum cost cell. The following example explains the method.

4.1 Example 3

India is a vast country. Sourcing managers will have surplus suppliers and have to be very decisive in choosing the right supplier. A valve manufacturing company has three suppliers to source Wafer Check Valves of certain specification and has to supply to three of its assembly units for further processing. The transportation cost is tabulated. Plan a suitable allocation that will minimize the transportation cost using lowest cost entry method.

Table 16. Transportation Cost and Supply Demand Data

	Destination			**Supply**
	1	**2**	**3**	**Capacity**
	(INR)			**(Qty)**
S1	60	40	10	**500**
S2	30	80	70	**400**
S3	40	40	20	**600**
Demand (Qty)	**200**	**950**	**350**	**1500**

4.2 Solution for example 3

Here in this case, Supply = Demand = 1500 Units. In this method, the cell having the lowest cost is first operated and the corresponding supply and demand corresponding to that particular cell is compared and whichever is lower is allocated. If in the case, demand is nullified, the entire corresponding row has to be eliminated and if supply is nullified, the entire column will be eliminated. If there are multiple cells containing the lowest cost, any one of the value can be chosen randomly. The following tables will explain the process of computing the BFS.

Table 17. Calculation of Initial BFS

	Destination			
	1	2	~~3~~	**S**
1	60	40	~~350~~ ~~10~~	500 – 350 =150
2	30	80	~~70~~	400
3	40	40	~~20~~	600
D	200	950	350 – 350 = 0	

Table 18. Calculation of Initial BFS

	Destination			
	~~1~~	2	~~3~~	**S**
1	~~60~~	40	~~350~~ ~~10~~	500 – 350 =150
2	~~200~~ ~~30~~	80	~~70~~	400 – 200 = 200
3	~~40~~	40	~~20~~	600
D	200 – 200 = 0	950	350 – 350 = 0	

Table 19. Calculation of Initial BFS

	Destination			
	~~1~~	2	~~3~~	S
~~1~~	~~60~~	~~150~~$_{40}$	~~350~~$_{10}$	500 – 350 =150 150 – 150 = 0
2	~~200~~$_{30}$	80	~~70~~	400 – 200 = 200
~~3~~	~~40~~	~~600~~$_{40}$	~~20~~	600 – 600 = 0
D	200 – 200 = 0	950 – 150 = 800 800 – 600 = 200	350 – 350 = 0	

Table 20. Calculation of Initial BFS

	Destination			
	~~1~~	2	~~3~~	S
~~1~~	~~60~~	~~150~~$_{40}$	~~350~~$_{10}$	500 – 350 =150 150 – 150 = 0
2	~~200~~$_{30}$	200_{80}	~~70~~	400 – 200 = 200 200 – 200 = 0
~~3~~	~~40~~	~~600~~$_{40}$	~~20~~	600 – 600 = 0
D	200 – 200 = 0	950 – 150 = 800 800 – 600 = 200 200 – 200 = 0	350 – 350 = 0	

Here all columns and rows have been now nullified.
Transportation Cost = [(30 × 200) + (40 × 150) + (80 × 200) + (40 × 600) + (10 × 350)] = Rs. 55,500
The optimality of this solution has to be checked.

4.3 Non degeneracy test

Individual Allocations= (No.of sources) + (No. of destinations) - 1
In this case,
Individual allocations = 5, Number of sources = 3
Number of destinations = 3
Hence, 3+3-1 = 5 . Hence the condition is satisified

4.4 Cell evaluation test

Table 21. Calculation of Cost Components

	Destination			
	1	**2**	**3**	$\mathbf{x_i}$
1		40	10	**0**
2	30	80		**40**
3		40		**0**
$\mathbf{y_j}$	**-10**	**40**	**10**	

The x_i and y_j components for each allocated cell are calculated as follows. Initially assume $x_1 = 0$

$c_{12} = x_1 + y_2 = 40 \rightarrow y_2 = 40$
$c_{13} = x_1 + y_3 = 10 \rightarrow y_3 = 10$
$c_{21} = x_2 + y_1 = 30 \rightarrow y_1 = -10$
$c_{22} = x_2 + y_2 = 80 \rightarrow y_2 = 40$
$c_{32} = x_3 + y_2 = 40 \rightarrow y_2 = 0$

For the solution to be optimal,

$$[m_{ij}] \geq 0 \text{ where, } [m_{ij}] = [c_{ij}] - [x_i + y_j]$$

Where, $[c_{ij}]$ = cost matrix of unallocated cells

In this case,

$$[c_{ij}] = \begin{bmatrix} 60 & 70 \\ 40 & 20 \end{bmatrix} \;\&\; [x_i + y_j] = \begin{bmatrix} -10 & 50 \\ -10 & 10 \end{bmatrix}$$

$$[m_{ij}] = = \begin{bmatrix} 70 & 20 \\ 50 & 10 \end{bmatrix} > 0$$

Hence the optimal solution is achieved.

<u>Suggested allocation</u>

1. Source 1 to Destination 2 : 150 units
2. Source 1 to Destination 3: 350 units
3. Source 2 to Destination 1 : 200 units
4. Source 2 to Destination 2 : 200 units
5. Source 3 to Destination 2 : 600 units

Minimum transportation cost that will be incurredthrough this allocation = Rs. 55,500

5.0 Calculation of Minimum Transportation Cost Using MODI method

Modified Distribution Method (MODI) method is an iterative technique wherein all allocations are modified witheach iteration until optimality is reached. Real time case which fails to solve under Lowest Cost Entry method and NW Corner Method (as in Example 01) can be proceeded further with MODI method until an optimal solution is reached.

5.1 Example -4

Plan a suitable allocation to source 10,000 units of Manual Pallet Valve from three sources and to be distributed to 4 destinations. The transportation costs have been tabulated. Use MODI method of transportation.

Table 22. Transportation Cost and Supply Demand Data

	Destination				**Supply Capacity**
	1	**2**	**3**	**4**	
	(INR)				**(Qty)**
S1	30	10	70	40	**2500**
S2	20	60	50	90	**3500**
S3	80	30	30	20	**4000**
Demand (Qty)	**2000**	**3000**	**3500**	**1500**	**10,000**

5.2 Solution for example -4

Here in this case Supply = Demand = 10,000 Units.Initial BFS can be obtained either by NW Corner or by Lowest cost entry method. In this case, NW Corner method has been used.

Table 23. Calculation of Initial BFS

	Destination				
	~~1~~	**~~2~~**	**3**	**4**	**S**
~~1~~	~~2000~~ $_{30}$	~~500~~ $_{10}$	~~70~~	~~40~~	**2500 – 2000 = 500 500 – 500 = 0**
~~2~~	~~20~~	~~2500~~ $_{60}$	~~1000~~ $_{50}$	~~90~~	**3500 – 2500 = 1000 1000 – 1000 = 0**
3	~~80~~	~~30~~	~~2500~~ $_{30}$	1500 $_{20}$	**4000 –**

					2500 = 1500 1500 – 1500 = 0
D	**2000 – 2000 =0**	**3000 – 500 = 2500 2500 – 2500 = 0**	**3500 – 1000 = 2500 2500 - 2500 = 0**	**1500 – 1500 = 0**	

Total transportation cost = [(2000 × 30) + (500× 10) + (2500 × 60) + (1000 × 50) + (2500 × 30) +(1500 × 20)] = Rs 3,70,000

5.3 Optimality test - non degeneracy test

For the solution to be non-degenerate, it must obey the following equation,

Individual Allocations = (No.of sources) + (No. of destinations) - 1

In this case,Individual allocations = 6, Number of sources = 3

Number of destinations = 4

Hence, 4+3-1 = 6. Hence the condition is satisfied

5.4 Cell evaluation test

In this test, the cost coefficients of the allotted cells are split into x_i and y_j components. The x_i and y_j components for each allocated cell are calculated as follows. Initially assume $x_1 = 0$ and the same procedure in the previous examples has been adopted.

For the solution to be optimal,

$[m_{ij}] \geq 0$, where,**$[m_{ij}] = [c_{ij}] - [x_i + y_j]$**

Table 24. Calculation of Cost Components

	Destination				
	1	**2**	**3**	**4**	**x_i**
1	30	10			**0**
2		60	50		**50**
3			30	20	**30**
y_j	**30**	**10**	**0**	**-10**	

Where, **$[c_{ij}]$ =** cost matrix of unallocated cells

In this case,

$$[c_{ij}] = \begin{bmatrix} & & 70 & 40 \\ 20 & & & 90 \\ 80 & 30 & & \end{bmatrix} \;\&\; [x_i + y_j] = \begin{bmatrix} & & 0 & -10 \\ 80 & & & 40 \\ 60 & 40 & & \end{bmatrix}$$

$$[m_{ij}] = \begin{bmatrix} & & 70 & 50 \\ -60 & & & 50 \\ 20 & -10 & & \end{bmatrix} < 0$$

Hence the solution is not optimal. Hence the solution has to be iterated.

5.5 Iteration -1

In this step, the key cell matrix has to be computed.

Key Cell Matrix = - $[m_{ij}]$

In this case

$$\text{Key cell matrix} = \begin{bmatrix} & & -70 & -50 \\ 60 & & & -50 \\ -20 & 10 & & \end{bmatrix}$$

The most positive element is the key-cell. Here the most positive element is 60. Hence the cell corresponding to the cell is cell 21.

Hence this cell will be held as key-cell. From the key cell a closed circuit has to be constructed through the allocated cells. Alternate positive and negative signs have to be assigned to the cells which lie on the closed circuit. In the closed circuit, the least quantity of negative allocation is noted and the value is either added or subtracted from the cell allocation value based on the assigned polarity. The following steps will explain the process of reallocation.

Table 25. Reallocation of Resources.

2000$_{30}$ (-) ↓	←**500**$_{10}$ (+)	70	40
20 (+)	↑ **2500**$_{60}$	50	90

(Key cell) →	(-)		
80	30	30	20

Table 26. Reallocation of Resources.

2000 – 2000 = 0 30	**500 + 2000 = 2500** 10	70	40
0 + 2000 = 2000 20	**2500 – 2000 = 500** 60 (Key cell)	50	90
80	30	30	20

5.6 Cell Evaluation Test – Iteration -1

Cost coefficients of the allotted cells are split into x_i and y_j components as per the usual procedure.

Table 27. Calculation of Cost Components

	Destination				x_i
	1	**2**	**3**	**4**	
1		10			**0**
2	20	60	50		**50**
3			30	20	**30**
y_j	**-30**	**10**	**0**	**-10**	

For the solution to be optimal,

$$[m_{ij}] \geq 0, \text{ where, } [m_{ij}] = [c_{ij}] - [x_i + y_j]$$

Where, $[c_{ij}]$ = cost matrix of unallocated cells

In this case,

$$[c_{ij}] = \begin{bmatrix} 30 & & 70 & 40 \\ & & & 90 \\ 80 & 30 & & \end{bmatrix} \& \ [x_i + y_j] = \begin{bmatrix} -30 & & 0 & -10 \\ & & & 40 \\ 0 & 40 & & \end{bmatrix}$$

$$[m_{ij}] = = \begin{bmatrix} 60 & & 70 & 50 \\ & & & 50 \\ 80 & -10 & & \end{bmatrix} < 0$$

Hence the achieved solution is not the optimal one. Hence the allocation needs to be iterated again.

5.7 Iteration 2

Key Cell Matrix = - $[m_{ij}]$

In this case

$$\text{Key cell matrix} = \begin{bmatrix} -60 & & -70 & -50 \\ & & 0 & -50 \\ -80 & 10 & & \end{bmatrix}$$

In this case the key cell is cell 32.

Table 28. Reallocation of Resources.

30	**2500**$_{10}$	70	40
20	**500**$_{60}$ (-) ↓	←**1000**$_{50}$ (+)	90
80	30 (+) Key Cell →	↑ **2500**$_{30}$ (-)	20

Table 29. Reallocation of Resources.

30	**2500** 10	70	40
2000 20	60 (Key cell)	**1000 + 500 = 1500** 50	90
80	**0+ 500 = 500** 30	**2500 - 500 = 2000** 30	**1500** 20

5.8 Cell evaluation test – iteration 2

Cost coefficients of the allotted cells are split into x_i and y_j components as per the usual procedure.

Table 30. Calculation of Cost Components

	Destination				x_i
	1	**2**	**3**	**4**	
1		10			**0**
2	20		50		**40**
3		30	30	20	**20**
y_j	**-20**	**10**	**10**	**0**	

For the solution to be optimal,

$$[m_{ij}] \geq 0 \text{ where, } [m_{ij}] = [c_{ij}] - [x_i + y_j]$$

Where, $[c_{ij}]$ = cost matrix of unallocated cells

In this case,

$$[c_{ij}] = \begin{bmatrix} 30 & 70 & 40 \\ & 60 & 90 \\ 80 & & \end{bmatrix} \& \; [x_i + y_j] = \begin{bmatrix} -20 & 10 & 0 \\ & 50 & 40 \\ 0 & & \end{bmatrix}$$

$$[m_{ij}] = = \begin{bmatrix} 50 & 60 & 40 \\ & 10 & 50 \\ 80 & & \end{bmatrix} > 0$$

Hence the achieved solution is the optimal one.

Suggested allocation

1. Source 1 to Destination 2: 2500 units
2. Source 2 to Destination 1: 2000 units
3. Source 2 to Destination 3 : 1500 units
4. Source 3 to Destination 2 : 500 units
5. Source 3 to Destination 3 : 2000 units
6. Source 3 to Destination 4: 1500 units

Minimum transportation Cost that will be achieved through this allocation = Rs. 2,45,000

6.0 Calculation of Minimum Transportation Cost for Unbalanced Demand and Supply

When the demand and the supply are unbalanced, a dummy row or a dummy column has to be added accordingly. Adding a dummy row or dummy column will convert the inequality to equality. This can be explained by an example.

6.1 Example-5

Obtain the Initial BFS for the unbalanced transportation case shown below.

Table 31. Transportation Cost and Supply Demand Data

	Destination				Supply Capacity
	1	2	3	4	
	(INR)				(Qty)
S1	30	80	90	150	300
S2	20	30	80	70	100
S3	60	90	70	70	150
S4	20	10	10	90	50
Demand (Qty)	100	150	170	100	520/600

6.2 Solution for example 05

In this case Demand and Supply are unequal. Hence a dummy column has been added to balance the case. In this case NW Corner method has been initially used.

Table 32. Calculation of Initial BFS

	Destination				Dummy	S
	~~1~~	~~2~~	~~3~~	4		
~~S1~~	~~100~~ ~~30~~	~~150~~ ~~80~~	~~50~~ ~~90~~	~~150~~	~~0~~	300–100=200 200–150=50 50–50=0
~~S2~~	~~20~~	~~30~~	~~100~~ ~~80~~	~~70~~	~~0~~	100–100=0

~~S3~~	~~60~~	~~90~~	~~20~~ ~~70~~	~~100~~ ~~70~~	~~30~~ ~~0~~	150 – 20 = 130 130 – 100 = 30 30 – 30 = 0
S4	~~20~~	~~10~~	~~10~~	~~90~~	50 0	50 – 50 = 0
D	100 – 100 = 0	150 – 150 = 0	170 – 50 = 120 120 – 1	100 – 100 = 0	80 – 30 = 50 50 – 50 = 0	

			00 = 20 20 – 20 = 0			

Total transportation cost = [(30 × 10) + (90× 50) + (80× 150) + (80× 100) + (70× 20) +(70× 100) + (30 ×0) + (50 × 0)] = Rs 35,900

This solution can be checked for optimality and can be further improved using the same procedure as in previous examples.

7.0 Conclusions

Various methods to estimate the suitable allocation and to compute the minimum transportation cost has been discussed. These simple techniques when applied to sourcing and expediting operations will lessen the work efforts and will achieve a state of managerial excellence.

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